Production mechanisms of open heavy flavor mesons and quarkonia in high-multiplicity events

Marat Siddikov, Iván Schmidt
Departamento de Física, Universidad Técnica Federico Santa María,
y Centro Científico - Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile

In this proceeding we discuss different mechanisms of open-heavy flavor meson and heavy quarkonia production which contribute to inclusive and single diffractive (SD) cross-sections. For the case of inclusive production of open heavy flavor mesons, we evaluated explicitly the contributions of the two-Pomeron and the three-Pomeron fusion. We found that the latter mechanism is significant for the D-meson production in the kinematics of small transverse momenta p_T and helps to improve the description of experimental data. Its role is less important at larger p_T, as well as for B-mesons. In order to confirm the role of the three-pomeron mechanism, we also analyzed the contribution of the single diffractive production of the open heavy flavor mesons. We found that it constitutes 0.5–2 percent of the inclusive cross-sections, yet this smallness is entirely due to the so-called gap survival factors which exist in single-diffractive case. Our theoretical results for SD production are in reasonable agreement with the available experimental data from Tevatron, and theoretical predictions of other approaches. Our estimates indicate that in LHC kinematics the cross section is sufficiently large for experimental studies. The expected dependence on event multiplicity in this channel is significantly milder than for inclusive production. Finally, for the quarkonia states we analyzed different production mechanisms for S-wave and P-wave quarkonia, and evaluated the expected multiplicity dependence. We argue that a rapidly growing multiplicity dependence seen in from STAR and ALICE collaborations presents a strong evidence in favor of multigluon fusion mechanisms in production of J/ψ and D-mesons. In order to confirm these findings we suggest to extend studies of multiplicity dependence to other quarkonia: we expect that other $1S$-quarkonia states, such as $\psi(2S)$ and $\Upsilon(1S)$, the multiplicity dependence should be close to that of J/ψ, whereas for $1P$ quarkonia states (χ_c, χ_b mesons) the multiplicity dependence should be milder than that of $1S$ quarkonia. We expect that the experimental confirmation of this result could constitute an important test of our understanding of multiplicity enhancement mechanisms in the production of different heavy mesons.

I. INTRODUCTION

Nowadays the production of heavy mesons play the central role in understanding the dynamics of strong interactions in nonperturbative regime, since in the abstract limit of the infinitely heavy quark mass the interaction in essence becomes perturbative. While in general this approach gives reliable and self-consistent description (see *e.g.* [1–10]), it is known that sometimes the description might require inclusion of nonperturbative components, like for example the fragmentation functions of open heavy flavour mesons and Long Distance Matrix Elements (LDMEs) of quarkonia states [6–12]. These new phenomenological observables as of now cannot be extracted from the first principles, and thus are extracted from analysis of the global fits of various available data. In view of complexity of these objects and their large number, it is always desirable to extend the list of observables which might be used for analysis, and fortunately experimental advances allow to proceed in this direction. One of the observables which recently got in focus of theoretical and experimental studies is the dependence of the cross-section on multiplicity of hadrons co-produced together with a heavy meson [13–19]. Since lack of statistics presents the largest obstacle for measurement of rare high-multiplicity processes, it is expected that analyzed multiplicity studies could benefit from enhanced luminosity in Run 3 at LHC (HL-LHC mode) [22–24].

Due to Local Parton Hadron Duality the multiplicity of light hadrons is proportional to the multiplicity of partons in collision, so the dependence on multiplicity allows us to estimate the density of partons (mostly gluons) with different rapidities during a collision. Since the multiplicity dependence in each gluon cascade (pomeron) which participates in collision is known from theory, the experimental measurement of multiplicity dependence of different process might allow to test the validity of the pomeron-pomeron (gluon-gluon) fusion paradigm, as well as onset of saturation due to higher twist multipomeron interactions. As was found recently by STAR and ALICE [13–18], for the production of J/ψ and D-mesons the multiplicity dependence is slightly faster than could be expected from pomeron-pomeron fusion. We argue that one of the possible explanations of this effect could be sizeable contribution of multipomeron mechanisms (e.g. triple pomeron mechanisms). While theoretical estimates in the heavy quark mass limit suggest that such corrections should be suppressed as higher twist effects, due to increase of gluon densities they might give sizeable contribution, thus challenging all the evaluations in the k_T- and collinear factorization approach. Eventually for very small values of x it is expected that they will lead to onset of saturation regime. In this kinematics consideration based on partonic picture might be not reliable at all, and it is more appropriate to use the frameworks with buit-in saturation, like *e.g.* the Color Glass Condensate approach and closely related color dipole picture [25–33]. In eikonal picture, which is valid at high energy, the color dipoles become eigenstates of interactions, and for this reason might be treated as universal objects which automatically accumulate all possible interactions of dipoles with the target [34]. The cross-sections of different processes in this approach might be expressed in terms of the color singlet dipole scattering amplitude known from Deep Inelastic Scattering. This framework gives successful description of the cross-sections of various lepton-hadron and hadron-hadron processes [35–42]. The generalization of the approach to high-multiplicity events is straightforward and was discussed in [43–50].

For the heavy meson production, the color dipole cross-section might be expressed via forward dipole amplitudes, convoluted with the wave functions of quarkonia or fragmentation functions of the open heavy flavour mesons. The exact structure of the expressions depends on the process under consideration. A central assumption implicitly used for the derivation of such relations is the dominance of the two-pomeron fusion mechanism, which eventually leads to the fact that the cross-sections of inclusive processes, as well as amplitudes of exclusive processes are expressed as *linear* superposition of forward dipole amplitudes. While the dipole amplitudes might include certain higher twist corrections due to multigluon exchanges, the use of the above-mentioned scheme only partially takes into account such corrections. A systematic evaluation of higher twist corrections might lead to nonlinear terms in relations between different dipole amplitudes. An example of such corrections, the so-called *quadrupole* contributions studied in [75], eventually lead to additional quadratic (bilinear) terms in the reation between inclusive production cross-sections and dipole amplitudes. In case of heavy meson production it is possible to estimate the role of such contributions numerically. Their role becomes especially important for description of high-multiplicity events, since such corrections become enhanced compared to linear terms.

In the following sections of this proceedings we briefly summarize the main results for the cross-section of different heavy mesons (both quarkonia and open heavy flavour mesons), and mke numerical estimates of the three-pomeron contributions. In the Section III we discuss extesnion of the framework for description of high multiplicity events, make proper numerical predictions, and wherever available, compare with existing experimenta data. Finally, we summarize and draw conclusions in the last Section IV.

II. PRODUCTION OF HEAVY HADRONS

As we mentioned in the introduction, the color dipole framework allows to express the cross-sections of various processes via forward dipole amplitudes, convoluted with the wave functions of quarkonia or fragmentation functions of the open heavy flavour mesons. The exact structure of the expressions depends on the process under consideration. The cross-section of heavy flavour production might be written as [5, 50, 51]

$$\begin{aligned}\frac{d\sigma_{pp\to\bar{Q}_iQ_i+X}\left(y,\,\sqrt{s}\right)}{dy\,d^2p_T} &= \int d^2k_T x_1\,g\left(x_1,\,\boldsymbol{p}_T-\boldsymbol{k}_T\right)\int_0^1 dz \qquad (1)\\ &\times\int\frac{d^2r_1}{4\pi}\int\frac{d^2r_2}{4\pi}e^{i(r_1-r_2)\cdot\boldsymbol{k}_T}\,\Psi_{\bar{Q}Q}^\dagger\left(r_2,\,z,\,p_T\right)\Psi_{\bar{Q}Q}^\dagger\left(r_1,\,z,\,p_T\right)\\ &\times N_M\left(x_2(y);\,\boldsymbol{r}_1,\,\boldsymbol{r}_2\right)+\left(y\to-y\right),\\ x_{1,2}&\approx\frac{\sqrt{m_M^2+\langle p_{\perp M}^2\rangle}}{\sqrt{s}}e^{\pm y} \qquad (2)\end{aligned}$$

where the variables y and $\boldsymbol{p}_T$ are the rapidity and transverse momenta of heavy quark, z is the light-cone fraction of the quark in the dipole, and $(\boldsymbol{r}_1,\,\boldsymbol{r}_2)$ are the transverse sizes of a dipole in the amplitude and its conjugate. The notation $\Psi_{\bar{Q}Q}(r,\,z)$ stands for the light-cone wave function of the $\bar{Q}Q$ pair of transverse size r and the light-cone fraction of the quark z (we'll use perturbative expressions from [57] for it). For the quarkonia production the cross-section is given by a similar expression

$$\begin{aligned}\frac{d\sigma\left(Y,\,Q^2\right)}{dy\,d^2\boldsymbol{p}_T} &= \int d^2k_T x_1\,g\left(x_1,\,\boldsymbol{p}_T-\boldsymbol{k}_T\right)\int_0^1 dz_1dz_2\int\frac{d^2r_1}{4\pi}\,\frac{d^2r_2}{4\pi}\,d^2b\;e^{-i\boldsymbol{k}_T\cdot\boldsymbol{b}}\times \qquad (3)\\ &\times\left\langle\Psi_{\bar{Q}Q}^\dagger\left(r_1,\,z_1\right)\,\Psi_M\left(r_1,\,z_1\right)\right\rangle\left\langle\Psi_{\bar{Q}Q}^\dagger\left(r_2,\,z_2\right)\,\Psi_M\left(r_2,z_2\right)\right\rangle\times\\ &\times N_M\left(x_2(y);\,\boldsymbol{r}_1,\,\boldsymbol{r}_2\right)+\left(y\to-y\right)\end{aligned}$$

where the variable $\boldsymbol{b}$ is a difference of the impact parameters of the dipole in the amplitude and its conjugate, Ψ_M is the light-cone wave function of the quarkonium, and all the other notations are the same as in (1). Both expressions (1) and (3) include a nonperturbative object N_M which encodes interactions of a dipole with the target. The structure

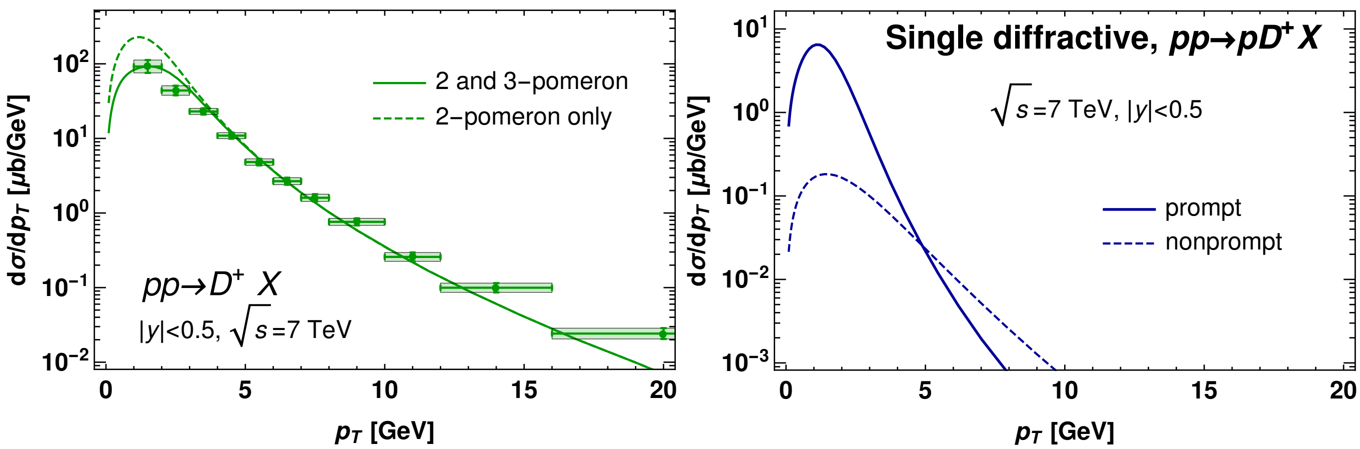

Figure 1. The D^+-meson production cross-section as a function of transverse momentum p_T. Left and right plots correspond to inclusive and semidiffractive production at central rapidities respectively. As explained in the text, the three-pomeron contribution in left-hand side is not positively defined due to interference diagrams, so its inclusion *decreases* the overall result. The experimental data are from [65–67].

of this object depends on the quantum numbers of the dipole and production mechanism. For the case of inclusive and semidiffractive production it might be found in [5, 20, 51, 52, 68]. In general this object is expressed as a linear superposition of the color singlet dipole amplitude $N(x,\, \boldsymbol{r},\, \boldsymbol{b})$ for the two-pomeron mechanism, and superposition of bilinear (quadratic) terms for the three-pomeron contributions. In general the three-pomeron contributions are not positively defined and cannot be interpreted probabilistically. In what follows we will use the CGC parametrization of the singlet color singlet dipole amplitude available from [58].

In the Figure 1 and 1 we have shown some of our results predictions for the inclusive and semidiffractive production in the central kinematics. The semidiffractive contribution is suppressed stronger at large p_T since the dominant contribution to it is formally a higher twist effect. We found that description of B-mesons and nonprompt J/ψ meson production in dipole approach has the same level of agreement with data (see [20, 51, 52, 68] for details), so we may use this approach for studies of multiplicity.

In general we can see that the color dipole approach gives reasonable estimates for the cross-sections, so we can use it for studies of multiplicity dependence.

III. MULTIPLICITY DEPENDENCE

In this section we the results for the multiplicity dependence of the hevay meson cross-sections. In the literature these results are conventionally given for the self-normalized ratio [70]

$$\frac{dN_M/dy}{\langle dN_M/dy\rangle}\;=\frac{d\sigma_M\left(y,\,\eta,\,\sqrt{s},\,n\right)/dy}{d\sigma_M\left(y,\,\eta,\,\sqrt{s},\,\langle n\rangle=1\right)/dy}\Bigg/\frac{d\sigma_{\rm ch}\left(\eta,\,\sqrt{s},\,Q^2,\,n\right)/d\eta}{d\sigma_{\rm ch}\left(\eta,\,\sqrt{s},\,Q^2,\,\langle n\rangle=1\right)/d\eta}, \tag{4}$$

where $n=N_{\rm ch}/\langle N_{\rm ch}\rangle$ is the relative enhancement of multiplicity, the variables y and η stand for the pseudorapidities of heavy meson and charged partcles respectively. The numertator and denominator of (4) correspond to relative multiplicity dependence of the heavy meson (M) hadroproduction and that of the inclusive channel.

As is known from the literature, [43–49], the saturation scale Q_s^2 in high-multiplicity events increases as

$$Q_s^2\left(x,\,b;\,n\right)\approx n\,Q^2\left(x,\,b\right). \tag{5}$$

For studies of the multiplicity dependence it is important to mention that the prescription (5) should be applied only to the dipole amplitude if the multiplicity enhancement is measured in the rapidity interval between a heavy dipole and the target. Application of such prescription is straightforward in case of single-diffractive production, as well as inclusive production in special kinematics when charged particles and heavy mesons are collected in bins well-separated kinematically. In the other widely used setup with overlapping bins in η and y, the enhanced multiplicity is shared by all the reggeized gluons which partcipate in the process, namely, leads to modification of gluon density as well as dipole amplitude in (1, 3). In the Figures we show our expectations for the multiplicity dependence of

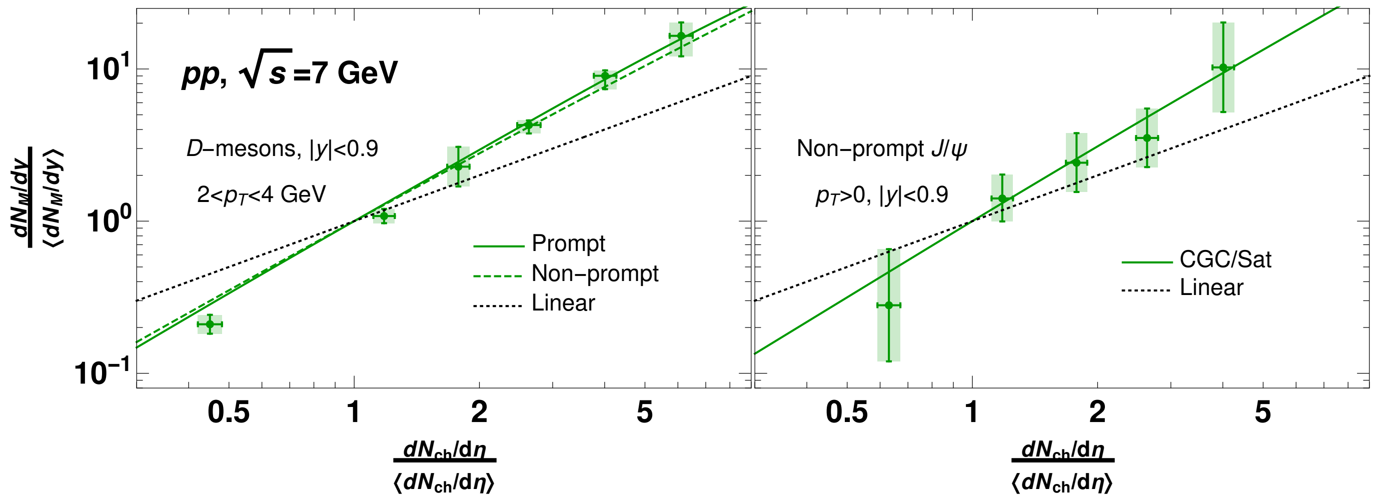

Figure 2. Left: Comparison of theoretical expectations and experimental results for the multiplicity dependence of D-meson production. Right: Similar plot for the multiplicity dependence of the non-prompt J/ψ quarkonia . In both plots the experimental data are taken from ALICE [13], and it is assumed that all particles are collected at central rapidities in rapidity window $|y|$, $|\eta|$ shown in legends of each plot. A short-dashed line in both plots corresponds to a linear dependence $(dN_M/dy)/\langle dN_M/dy\rangle \sim n$ and is added for the sake of reference.

the inclusive and single-diffractive data. Whenever possible, we also make comparison with experimental data. We can see that the model provides quite reasonable predictions for the inclusive production. The contributions of the nonprompt mechanisms (shown by dashed line) has n-dependence similar to that of prompt contribution. For the single diffractive production the dependence on multiplicity is milder than for inclusive production due to specifics of this process (in pomeron language, it has only one cut pomeron which contributes). The predictions for the B-mesons and nonprompt J/ψ mesons behave in a similar way and are available from [20, 51, 52, 68].

IV. CONCLUSIONS

In this proceeding we analyzed in detail different production mechanisms of heavy mesons (namely D-, B-mesons and $1S$ quarkonium state J/ψ). We found that there are sizeable contributions from higher twist three-pomeron mechanism. This contribution becomes especially pronounced for charm sector at small transverse momenta p_T, where it decreases the cross-section by up to forty percent. We found that inclusion of this correction gives better description of experimental data and helps to describe the multiplicity dependence without any additional assumptions. This contribution is less important for the B-mesons, as well as at large transverse momenta $p_T \gtrsim 10$ GeV, in agreement with general expectations based on twist counting. We expect that the three-pomeron contribution also will significantly modify p_T-integrated observables, which get dominant contribution from the small-p_T-region.

We found that inclusion of the three-pomeron correction allows to describe the experimentally measured multiplicity dependence, both for open heavy flavour mesons and $1S$ quarkonia. In order to verify experimentally the importance of this mechanism in quarkonia production, we suggest to study the production of $1P$ states (χ_c and χ_b mesons). Since for these states the three-pomeron fusion is strongly suppressed, the production cross-section for these states should have much milder multiplicity dependence.

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