
Report of the First Referee

> The article entiteled "Ground-state and thermodynamic properties of the spin-1/2 Heisenberg model on the triangular lattice" by Gonzalez and collaborators uses high-temperature series expansions in combination with the entropy method, which is a specific extrapolation tool, to calculate and analyze the ground-state energy and the specific heat of the Heisenberg model on the anisotropic triangular lattice. The latter includes as limiting cases the 1d chain, the square lattice, and the isotropic triangular lattice, which are all well known from previous analytical and numerical studies. In addition, extensions of the model including spatial anisotropies, next-nearest neighbor Heisenberg interactions, and non-magnetic impurities are also considered which are in particular motivated by experimental findings on the frustrated quantum magnet Ba_8CoNb_6O_24. In my opinion the article is well written and the topic is interesting. Globally, I therefore recommend publication in SciPost. I nevertheless have some points, which the authors should address to further improve their manuscript.

We thank the First Referee for considering our article as well written and the topic as interesting. We are also thankful for the recommendation to publish it, and we address their suggestions and questions below.

> List of points:
>
> - Title: I wondered whether one should put "anisotropic triangular lattice" in the title which expresses better the focus of the article

The Referee is right about this point. We have renamed the article as: “Ground-state and thermodynamic properties of the spin-$\frac{1}{2}$ Heisenberg model on the anisotropic triangular lattice”

> - Figure 3, page 7: Is there a microscopic reason why a "1/n^2" scaling is used? If yes, maybe one can specify the argument. If not, one should mention it.

This is an interesting question. We tried several scaling types and kept the one that was working better. However, we do not have any microscopic argument to support it.
To clarify this point, we have added the previous sentence in the new version, on Page 6.

> - Page 8: There are two equations in the text which are too wide.

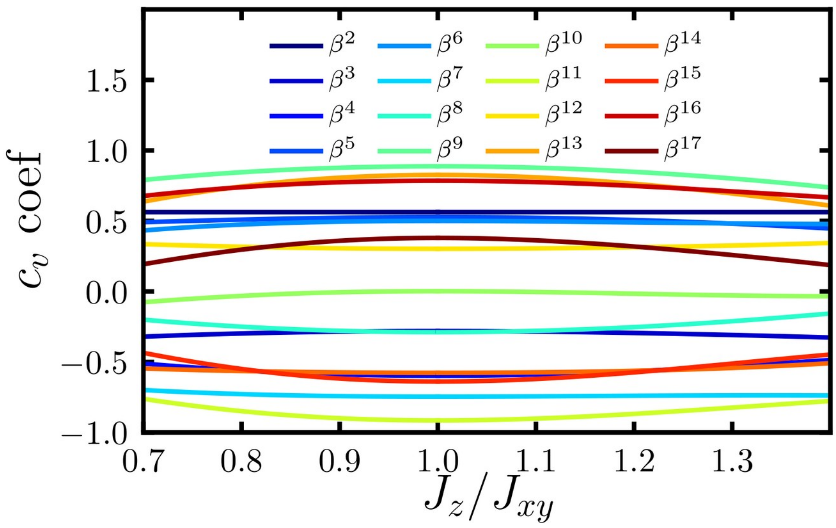
We thank the Referee for pointing out this formatting issue. It appears only through the arxiv compilation. We have solved it in the new version.

> - Page 9/10: Is there are a (physical) reason why the quality of the extrapolation seems worse on the square lattice compared to the triangular and chain limit? Naively I would have expected that the square lattice is the simplest system.

This is a very interesting question, regarding why the method works better or worse in different cases. However, we have no physical explanation. We can only speculate that this can be related to the $T=0$ behavior of the models, which is difficult to capture from the HTSE.

- Page 13: Is there a simple argument why the specific heat does not (almost does not) depend on the spin anisotropy?

First, it should be taken into account that the main effect of Jxy/Jz on the c_v has been removed by using the normalizing relationship 2Jxy^2+Jz^2=constant. The same thing happens in the case of the kagome lattice (Fig. 3 in https://doi.org/10.1103/PhysRevB.101.140403). After considering that constraint, the simplest argument is that if the curves are the same it is because the HTSE are the same. In practice, it can be seen that the coefficients do not change much when changing the relationship Jxy/Jz (when the constraint is fixed). In the figure below we show the values of all the coefficients in c_v(\beta) for different values of Jz/Jxy. It can be seen that the derivative of the coefficients with respect to Jz/Jxy is 0 around the isotropic point, so that small changes do not change the series. Furthermore, it can be seen that the coefficients do not change much in a larger range of Jz/Jxy values.



- Page 13: Another possibility/option could be the presence of multi-spin interactions. Can the relevance of such interactions be excluded for the considered material?

It is true that larger loop spin exchanges are important in simple models as He^3 or electron solids close to melting. Here what we call pair exchange or S_i.S_j is already a super exchange process going through oxygen orbitals (From Ref. [38]: “From the corner-sharing geometry of neighboring CoO6 and NbO6 octahedra, the dominant magnetic interactions between in-plane Co2+ spins occur by Co-O-O-Co and Co-O-Nb-O-Co superexchange couplings [17]; the very long paths make the interaction strength extremely sensitive to geometrical details and should preclude all but nearest-neighbor couplings.”). Thus larger loop exchanges will be a priori of much smaller importance. Also, for spin-1/2 considered here, three body loop exchanges can be exactly mapped on two-body exchanges. So, they are already taken into account, and it is unlikely that larger loop exchange would be of importance.

- Page 15: Title of reference [18] seems to be spoiled.

We thank the Referee for pointing that out. We have solved the issue in the revised manuscript.

We believe that, in the new version of the manuscript, we have answered properly all the questions and concerns pointed out by the Referee. So, we hope that they consider now that the present version is suitable for publication in SciPost Physics.