Response to Report 1

Referee Comment: *When deriving the spin-orbit coupling Hamiltonian in Sec.II-A, the authors should discuss if spin-orbit coupling term that are linear in k are symmetry allowed. In principle those term, if appearing, should be explicitly taken into account considering that the spin-orbit free Hamiltonian is expanded up to linear order in momentum.*

Our Response : We would like to point out that in addition to the k-independent spin-orbit coupling term, linear k-dependent terms are also allowed by symmetry. The corresponding terms are $\{k_x\sigma_1,\ k_y\sigma_2,\ k_y\sigma_3\}$ and $\{k_y\sigma_1,\ k_x\sigma_2,\ k_x\sigma_3\}$ which will appear as $\{\mathrm{H}^{12}(\mathbf{k}),\ \mathrm{H}^{34}(\mathbf{k})\}$ and $\{\mathrm{H}^{13}(\mathbf{k}),\ \mathrm{H}^{24}(\mathbf{k})\}$ elements respectively for eight-band $\mathbf{k}\cdot\mathbf{p}$ Hamiltonian blocks. They are given in Table-II in the main text. The linear k-dependent spin-orbit coupling term can be written in matrix form as

$$\mathcal{H}_{SO}^{k\cdot p} = \begin{pmatrix} 0 & 0 & A_1 & B_1 & A_1' & B_1' & 0 & 0 \\ 0 & 0 & B_1^* & -A_1 & B_1'^* & -A_1' & 0 & 0 \\ A_1 & B_1 & 0 & 0 & 0 & 0 & A_1' & B_1' \\ B_1^* & -A_1 & 0 & 0 & 0 & 0 & B_1'^* & -A_1' \\ A_1' & B_1' & 0 & 0 & 0 & 0 & A_1 & B_1 \\ B_1'^* & -A_1' & 0 & 0 & 0 & 0 & B_1^* & -A_1 \\ 0 & 0 & A_1' & B_1' & A_1 & B_1 & 0 & 0 \\ 0 & 0 & B_1'^* & -A_1' & B_1^* & -A_1 & 0 & 0 \end{pmatrix} \tag{1}$$

where $A_1 = \gamma_1 k_y$, $A_1' = \gamma_1' k_x$, $B_1' = \gamma_2' k_y - i\gamma_3' k_x$ and $B_1 = \gamma_2 k_x - i\gamma_3 k_y$ with $\gamma_1, \gamma_2, \gamma_1', \gamma_2', \gamma_3, \gamma_3'$ as SOC coupling strength.

We observe then that the linear-in-k terms couple the low-energy bands with the high-energy subspace only. When these are "integrated out", they can lead only to *quadratic* in k corrections to the SOC within the low-energy space. This observation is symmetry-based, and does not depend on any kind of approximation. In addition, we found that the dispersion near the Dirac node of monolayer WTe_2 with/without SOC obtained from the DFT analysis already matches extremely well with our low-energy model. Therefore, the contribution from the linear k-dependent SOC should be very small. Finally, the idea of dominant k-independent SOC agrees with the recent experimental results, as described in the paper. Given that all possible matrix elements for the 8-band model are already presented in the manuscript, we feel like have addressed this issue adequately.

Referee Comment: *In the same subsection, the authors also mention that there is conservation of the spin-projection in their effective Hamiltonian. Is this enforced by a crystalline symmetry, as it happens for instance in graphene when preserving the Mz horizontal mirror symmetry? Or this is an accidental property due to the assumptions made on the spin-orbit coupling terms?*

Our Response : We thank the referee for raising this point. The conservation of the spin-projection on a particular axis in the effective Hamiltonian actually stems from a combination of the crystalline symmetry, and band ordering in WTe_2. The latter makes the low-energy bands of the same parity, and then the crystalline symmetry implies that the leading SOC is momentum independent, making the spin projection onto d_s approximately conserved near the Γ point. The corrections to this picture are *quadratic* in k, and are small.

We have dedicated a paragraph to this issue: the one starting with "Corrections to the k-independent spin-orbit coupling (3) stem from other spin-flip hopping paths...", right before Section IIB. To make sure there is absolutely no possibility for confusion, we added a clarifying sentence to the end of that paragraph, stating that addition of sub-leading SOC would break spin conservation, but they don't.

Referee Comment: *A minor remark concerns the statement in Sec.II-B "There are several notable features of Hamiltonian. Each band that it describes is double degenerate at every k-point, as appropriate for a system with both time-reversal and inversion symmetry". I would not consider "notable" the double degeneracy of the bands in an inversion and time-reversal symmetric system.*

Our Response : We thank the referee for this suggestion. We have now modified the sentence in the revised manuscript.

Referee Comment: *I am confused by the expression for the anomalous Hall conductance due to planar magnetic field, that seems to be inversely proportional to the spin-orbit coupling strength (see Eq.25).The Berry curvature indeed grows proportionally with the SOC. Could the authors clarify the scaling and what happens to the transverse conductance in the spin-orbit free limit?*

Our Response : We note that we are talking about a situation near a band edge, where the gap - or the spin-orbit coupling - is the largest energy scale, so one could say that the Hall conductivity is inversely proportional to the gap, and that would be less surprising, perhaps. In more detail, the Berry curvature for the degenerate conduction bands in this system is given by (Eq. 24)

$$\Omega_z(\boldsymbol{k}) = -\frac{v_x v_y \Delta_{so}}{2\epsilon_{\boldsymbol{k}}^3}(\boldsymbol{\Sigma}\cdot\boldsymbol{d_s}). \tag{2}$$

In the low-energy limit, $\epsilon_{\boldsymbol{k}} \to \Delta_{so}/\sqrt{1-\beta^2}$ and therefore, the Berry curvature becomes $\Omega_z(\boldsymbol{k}) \propto 1/\Delta_{so}^2$.

To consider the limit of small SOC, one has to send the gap size to zero while keeping the Fermi level constant. Then for small enough gap, it will have only a perturbative effect on the Berry curvature at the Fermi level, and the AHE will simply vanish with vanishing SOC. This is not the situation we are interested in though.

Referee Comment: *It would be also beneficial to estimate the size of the Hall conductance for moderate magnetic fields and estimate if the effect can be in principle observed in experiments.*

Our Response : We thank the referee for this helpful suggestion. As is clear from Eq.(25), the magnitude of the Hall conductivity in units of the conductance quantum is set by the ratio of the Zeeman energy to the SOC strength. Hence it is fairly small, and is about 10^{-3} for moderate B-fields on the scale of 1T. This, however, is a perfectly measurable conductivity in the present-day experiment. For instance, this number exceeds by at least three orders of magnitude the precision of QHE quantization in strong magnetic fields. We added a note on the magnitude of the effect to the "Conclusions" section of the revised manuscript.