We appreciate your careful reading of our paper, raising many questions for us. Most of them seem to come from our poor explanation of the concepts and results. We try to improve them to be understandable for the referee and Sci-Post readers.

Report
In this numerical paper the authors consider a model of a classical particle which random movement in a finite D-dimensional cube of a size 2L_f in each direction is
mediated by the velocity field of a random dipole at the origin and numerically calculate the fractal dimension of the particle trajectory after
a finite number N of time steps of the fixed size δt.
The authors claim that the fractal dimension D_f of this trajectory as a D_f-dimensional object immersed in the D-dimensional space is smaller than
D and weakly depends on the system parameters: N, δt, L_f.
I) First of all, one can see that there are many references to the companion paper [23] (as well as to [24] in Sec. 6)
which makes the current numerical work not standalone and reduces the possibility to assess it independently.
I strongly recommend to merge two companion submissions into the one.

[Reply I]
Please refer to our opinion on Merge of papers, Model building, and Choice of parameters and Stable fractal dimension in [Main reply to Requested changes] which will be given at the end of this letter.

II) Second, the relevance of the model considered by the authors to the turbulence is doubtful, especially taking into account the suggested boundary conditions 1 and 2.
Indeed, following the discussion in Sec. 6.1 about the periodically located dipoles that should model the vortices in the periodically boundary condition 1, one can immediately see that
- All these "dipoles" are considered to be the copies of the same dipole with the completely correlated dipole moments for all of them.
- The dipolar fields are cut at the distance Lf from each dipole and do not penetrate to the next elementary cell. This makes all these dipoles to be short-range.In addition, in the introduction the relevance of the considered model with the "velocity field caused by a “dipole” with a randomly modulated moment located at the origin" should be motivated by the references in the literature.
For a moment, reading the introduction, one cannot understand how the only dipole at the origin of the infinite system can affect the dynamics
in 3D system where the probability of the return to this point is zero in the thermodynamic limit.
Please clarify all this in the introduction from the very beginning.

[Reply II]
We propose a simple toy model where a particle moves in a potential generated by a random dipole placed at the origin. Despite the simplicity, the trajectory of the particle has a fractal dimension. We propose that it may give a simple model of the fluid particle in hydrodynamic

turbulence.
1) We modified the sentence of [Sec 6 Discussion] as follows to avoid confusion by the readers. "We will give some theoretical prospects of various topics, which motivate the paper. While they are not within reach of this paper, they will be important in future research."
2) We erased typos regarding (Condition 1) and (Condition 2) in Sec. 6.1. Namely, we modified two sentences at the beginning of Sec. 6.1 as follows: "In this paper, we examined a toy model of hydrodynamics, where a randomly modulated dipole at the origin determines the velocity field in a finite box with edge length 2L_f. It models a picture that the emergence of eddies (or vortices) is responsible for the generation of turbulence. We simplified the eddies' modulation to that of a dipole in this paper.To investigate … Eqs. (15) and (17) in [23]. Then, …in 3D. (e.t.c.)"

III) The boundary condition 2 does not have any proper physical motivation from the turbulence. It is just the calculation of the fractal dimension of the object formed by the overlap of several independent particle trajectories started at the same initial point and escaped beyond the boundary of the D-dimensional cube. Please clarify the relevance of this boundary condition 2 to the physical system.

[Reply III]
We note that Mandelbrot adopted an analog of the Get-Back condition (Condition 2) in the study of fractal physics. He referred to it as resetting protocol. It became a standard technique in studying the fractal system. For clarification, we add an explanation in the paragraph after Condition 2.

IV) The authors consider a D-dimensional torus for the periodic boundary conditions. I wonder what will happen at the surfaces with different topology (genus): D-dimensional sphere of genus>1 surfaces?

[Reply IV]
If the stream channel includes obstacles, we have to consider the different topology other than the box for the stream channel. In D = 3, the genus does not exist, and we have to apply the geometrical decomposition theorem by Thurston and Perelman. In this paper, we include no obstacles so that the referee's question is beyond the scope of our paper.

V) In addition to the boundary conditions, one should carefully consider the definition of the fractal dimension, Eq. (8), and its dependence on the parameters.
The definition (8) of D_f is given by the fractal dimension of the trajectory as a D_f-dimensional object immersed in the D-dimensional space.
Unlike any static objects in an infinite D-dimensional space, the number of occupied boxes by a particle trajectory changes with the number of time steps N, with the system size L_f, with the boundary conditions and even with the time step δt.
As a result, the definition of the fractal dimension is highly non-universal and system-specific.

[Reply V]
Indeed, the trajectory of a fluid particle is not a static object, but it changes dynamically at each

time step N, as the referee argued. We note, however, that there is no "universal" definition of the fractal dimension, dimension, to be mentioned in [Main reply to Requested changes on Choice of parameters and Stable fractal dimension]. There should be a range of parameters where the fractal behavior is well-defined.
We computed the fractal dimension by the box-counting method to each trajectory at time step N. We note that the fractal dimension D_f attains a stable value, as indicated in Tables 1-4, and has a reasonably stable behavior of D_f on N and L_f as shown in Figure 3 and 4. Our claim of fractal dimensionality is the restricted one, whose validity is limited to the values written in the draft.
This implies that one may obtain the behavior of the trajectory for the other range of parameters, by rescaling the other parameters.

VI) Consequently, by using the above definition of D_f the authors have to consider the parameter dependence of D_f, which they partially did. Here I would like to ask several questions:
1. What is the reason to consider the concrete range of parameters (9-11)? How do the results change in the other range? What do the overlapping regions for
dh mean in (9)?

[Reply VI-1]
It is possible to interpret that Eqs. (9)–(11) give the range where the Levy flight condition holds. As we will mention in [Main reply to Requested changes on Choice of parameters and Stable fractal dimension], where one has a chance to observe the fractal behavior near the range, the following sentences were added to P.6 Sec. 3 subsections 'Overview of numerical results':
"Suppose the L'evy flight is realized by a big jump from the neighbor of the dipole to the neighbor of boundaries, we have approximately the [L'evy flight Condition (LfC)] as...".
Furthermore, the numerical data is re-collected to match the d_H parameter range, and Table 4 is updated.
Also, L'evy flight occurs under the same LfC when the parameter regions in which the L'evy flight occurs is changed.

2. How these range of parameters are related to the results in Tables 1-4?

[Reply VI-2]
The parameter region is determined by LfC as described above, and Table 1-4 shows the values for which the fractal dimension is considered to be the most characteristic of the system. The parameter regions also include regions that do not necessarily satisfy LfC, and in these regions the fractal dimension is closer to the value of an ordinary Gaussian random walk rather than this characteristic fractal dimension. In section 3, we add the following explanation:
"Suppose the L'evy flight...
little wider than this typical choice satisfying [LfC] ≈ 1."

3. What are the values of all the parameters (N, L_f, δL, initial point) in Tables 1-4?
In the current order, the results of the Tables 1-4 are unclear for a reader as they are given

before the ones from Sec. 4. Please consider to move the tables later.

[Reply VI-3]
We added the following sentences to P.6 Sec. 3, the subsection entitled 'Overview of numerical results': "In Table 1 to Table 4, $0.02 \leq d_H \leq 1.00$. In Table 1 to Table 4, $x0 = -0.2$," and "We will discuss the details in the next section. At first,..."

4. Why D_f=0 and σ=0 for dh>0.2 in Table 3? It is unclear from the table.

[Reply VI-4]
We agree that Table 3, which describes D_f and σ = 0.000 for d_H = 0.3 and 0.4, should be more definitely explained. While we mentioned it in the last sentence of the paragraph 'Overview of numerical results in section 3'', the sentence after "As detailed in the next section, ..." may not be appropriate. We modify it to by the following sentence:
"As detailed in the next section, the fractal dimension behaves in a similar pattern in 2D and 3D cases and the degree of decrease for D_f depend only on d_H. Especially in Table 3, we find D_f = 0.000 for d_H = 0.3 and 0.4. This shows that a phase transition-like effect exists for D_f by changing d_H. (See Eqs. (13) and (14)) It also occurs the decrease in particle numbers. It depends only on d_H. This phase transition-like effect is caused by particles being jumped out from the fundamental region at the next instant. If the behavior is analyzed in detail, δt is taken to be sufficiently small, which requires d_H to be taken to be inversely large."

5. The dependence of Df versus log10(N) given in Fig. 3 does not show any saturation. From the general point of view D_f should scale with N as the increase of the time steps increases the number of the boxes occupied by a trajectory. Therefore the statement "When the parameter N (the number of steps) varies ... the fractal dimension D_f asymptotically approaches a constant" is at least misleading.

[Reply VI-5]
We added the following explanation about the $\log_{10}$ (N) dependence at P.8 Sec.4 subsection 'Dependence on N, the number of steps'.
"You can find the $\log_{10}(N)$ dependence of the fractal dimension in Figure 3. Even if the scale invariance is not broken strongly by the power of N, it is weakly broken logarithmically in detail. We accept these logarithmic corrections and understand that the fractal dimension D_f is approximately constant.
We use the box-counting method here, but the need to increase the number of divisions as N increases is difficult from the standpoint of computation time when N is large enough. If the number of divisions cannot be increased sufficiently as N increases, the fractal dimension of the particle trajectory will asymptotically approach the spatial dimension since it covers the box. This is a technical issue, since if values of N are too large, then they do not give reliable results. For such N, calculations requires the larger number of divisions."

7. In the same way, D_f depends on the system size as by decreasing L_f(or increasing N) one can increase D_f to D for any dimensionality D.

[Reply VI-7]
As shown in Figure 4, the change in box size does not yield results that get closer to the corresponding spatial dimension as the size is reduced

8. Comparing Fig. 3 with the above statements, one cannot understand how the results for D_f in Tables 1-4 are obtained. As D_f cannot "asymptotically approach a constant, which is equal to the values given in the previous section", it is unclear
- which finite values of N and L_f are used in Tables 1-4 and why,
- what all this finite-size and finite-time values of D_f have in common with any fractal dimension of Levy flights and anomalous diffusion (e.g. to the one considered in [23]).

Please clarify the above statement of the manuscript about D_f and the definition of the fractal dimension used for a finite system size and a finite number of time steps.

[Reply VI-8]
We have added a note about the initial positions $x_0 = -0.2$, $y_0 = 0.0$, $z_0 = 0.0$ in Figures 3 and 4. See the answer to VI)-5 for a discussion of whether N approaches asymptotically.In the calculation of the box dimension, the number of points that fall within the box is counted while decreasing the box size, and the logarithmic graph of the values actually shows a straight line, so this is not an argument that looks only at finite size for the calculation of fractal dimension. Additionally, as in the calculation of fractal dimension in physical phenomena, there is a limit to the resolution, and we are not claiming that fractal geometry can be constructed down to the infinitely small scale.
For example, it is so for the Rias coast in nature.

9. The dependence on the initial location x_0 of the particle is also quite unclear.
Indeed, like in the item VIII) below, the entire Fig. 6 is relevant for the case of the missing particles of [23] (or partially to the boundary condition 2),
but not related to the periodic boundary conditions.
In this sense the phrase "the fractional dimension is not well-defined when the initial particle location is too close to the dipole" is misleading for the periodic boundary conditions 1.

[Reply VI-9]
We specified that we are using condition 2. The reason for using Condition 2 in the discussion of "Missing particle" is that this effect is more pronounced in Condition 2. This is because the effect of dependence on initial position becomes more visible when particles that have jumped out of the box size are returned to the same initial position and restarted.

10. The dependence of the critical position r_c on δt and L_f in (13) shows the strong dependence of the results both on δt and Lf which is neglected by the authors.
Please clarify this issue in the text.

[Reply VI-10]
As seen in (14), since r_c, δt, and L_f are interrelated, changing one of these values will change

the other. However, the relationship is not linear. We have added this discussion to Sec. 3 subsection 'Overview of numerical results, as [L'evy flight Condition (LfC)].

(Question)
11. The dependence of the results on the time step δt and of the size of the counting box δL is not considered by the authors.

[Reply VI-11]
The change in δt cannot be too small because errors accumulate with each additional numerical step. In addition, since the change in δt can be considered as the change in the dipole moment, δt is calculated as a fixed value.

VII) In addition, for solving continuous-time Eq. (6) the authors use the simple Euler integration scheme (7) with the fixed time step δt=0.01.
Why do not they use any Runge-Kutta integration scheme?
Does the Euler scheme conserve some integrals of motion?
What is the accuracy of Euler scheme?
How the results depend on the choice of δt=0.01?

[Reply VII]
Our equation to be solved here is an ordinary first-order differential equation. For such a simple equation, it is enough to use the Euler method. If we solve the partial differential equation such as the Navier-Stokes equation, which is beyond our papers, we may use the Runge-Kutta method, with the more sophisticated definition of difference operators. Also, since there is no substantial error, the Euler method is considered acceptable. About the choice of δt = 0.01, please refer to answer V) of this letter.

VIII) Figure 3 and the discussion around it is unclear.
Indeed, Eqs. (1-8) deal with a single particle, while the discussion of the section "Missing particles" includes "the number of particles".
It seems that the authors consider many random realizations of the same stochastic process and count the fraction of particles which cross the border of the D-dimensional cube of the size 2L_f.
This is especially unclear with respect to the boundary conditions 1 and 2 which do not have any missing particles by definition.
This part seems to be relevant for the companion manuscript [23] as there the fraction of missing particles in such random realizations is a relevant parameter of the probability loss.
This once again shows that the current submission should be merged with the one of [23].

[Reply VIII]
In this numerical calculation, the term "number of particles" is used in the following sense. When the number of steps and the number of trials are determined and the simulation of the trajectories is carried out stochastically, the “number of particles” means that the number of particles that stay in the box area, without counting the particles that leave the box area at that

number of steps. In other words, the statistics are obtained by repeating the process by the number of trials. Therefore, the choice of boundary condition 1 or 2 is irrelevant to the discussion of this calculation. Since this point is not clearly described in the paper, we have corrected it to make it clear. The following sentence has been added at the end of Sec. 4 Subsection 'Missing Particles':
"Therefore, the choice of Condition 1 or 2 is irrelevant to this calculation since we are only counting the number of particles out of the box."

IX) In addition to it, the authors claim that the above number of missing particles decreases exponentially in Fig. 3 (with the time step, I guess) which is not the case in [23], where Fig. 2 shows the approximately logarithmic decay (linear with log2(t)).This issue should be clarified.

[Reply IX]
Although it is difficult to easily compare the detailed behavior of the first and second papers due to their different analysis methods and parameter domains, it is nevertheless important to note here that both are consistent with a monotonically increasing number of missing particles.

To sum up, I cannot recommend the manuscript for the publication in SciPost Physics in the current form.
I may reconsider my decision if the authors address all my questions and comments, including the one about the merging with the companion submission (https://scipost.org/submissions/2201.04900v2/).

Requested changes
1 - Please merge this submission with [23] in order to make both projects consistent, standalone and results clear.

[Main reply to Requested changes on Merge of Papers]
You requested that we have to merge two papers, this one “Anomalous diffusion in a randomly modulated velocity Field” and the second one “Analytical Study of Anomalous Diffusion by Randomly Modulated Dipole” cited by [23]. We are very sorry not to accept your request, for the following reason: The first paper is a heuristic consideration of how a trajectory of a fluid particle becomes fractal, by using the numerical simulation on the stochastically modulated trajectory. The objective of this paper is to “find new facts” in our randomly modulated dipole model. On the other hand, the second paper attempts the theoretical understanding of our model by formulating the Fokker-Planck equation theoretically. The fractal dimension D_f adopted in the first and the second papers are different; in the first paper, the fractal dimension is defined by the Box Counting method, while in the second paper it is defined by the correlation functions. The first paper does not require difficult theoretical backgrounds, while the second paper requires a number of theories on the path integrals and quantization in the presence of the constraint. Therefore, the merging of two papers will be troublesome for the readers having different backgrounds.
In addition to this, we think there is no essential overlapping between the two papers.

You may mix up the model discussed in the main body and that in Sec. 6.1. The latter model is an addendum and never be used in the main body. This trouble seems to come from our typos; in the end of the first paragraph of Sec.6.1, the word (Condition 1) appears, but (Condition 1) and (Condition 2) in this section 6.1 are foreign to the conditions for the boundary condition mentioned in Sec.3.

2 - Please consider to change the description of the model from too generic one in (1-3) to the concrete one in (4-6):
The current description in (1) with randomly moving source locations ζi are confusing for a reader.
The only relevant parameter d(t) in (5-6) improves the clarity and the readability of the manuscript.

[Main reply to Requested changes on Model building]
You requested to omit the general situation having various sources and sinks and go directly to the single dipole, having a dipole moment d_H. However, in real turbulence, it occurs in some regions such as boundary layers and those after the obstacles. Therefore, we have to start from the general case and manifest our calculation is done in a simplified toy model with a single dipole moment, preserving the randomness of its modulation. Therefore, we keep Eqs. (1)-(3). We define, however, the dipole moment d_H more clearly, and we add sentences to make it clear why we take a special case with a single dipole.

Revision of Sec. 2: Model description
Before the sentence “We focus on a ...” in the above of Eq. (4), we add the following sentence, “As was discussed in the above, the real turbulent phenomena is a very complicated one, which involves many vortices (eddies) of different sizes and vorticity, or many sources and sinks with different quantities of fluid (charge) Q coming in and out per unit time.
Here, we consider a simple toy model, in which there exists a single dipole (with a single source and sink). More explicitly, the locations of the sink and source are identified, having a constant dipole moment, dH, but we keep the essential ingredient of random modulation which can be stated in other words, the magnitude of the dipole moment d_H = |**d**_H| is fixed time-independently, while the direction of the moment, **d**ˆ_H(t) = **d**_H(t)/d_H = is randomly (stochastically) modulated.
That is, we focus on ...”

3 - Please reconsider and clarify the definition and the parameter dependence of the fractal dimension (see the report).

[Main reply to Requested changes on Choice of parameters and Stable fractal dimension]
We will modify the manuscript as far as we can to clarify the parameter dependence of the fractal dimension at Reply Ⅵ) -1 and V)-2. Your observation that the fractal dimension

depends on various
parameters and the universality is violated for a finite time-interval and a finite space region is basically correct. In the turbulence phenomena, the scaling behavior appears frequently, but it can not be proved by solving the fundamental equation of hydrodynamics (Navier-Stokes equation). A more detailed study indicates that the naive scaling is violated and the turbulent fluid can be a multi-fractal system. Even this multifractality can not, however, be verified.

The important fact to happen here is that for a certain range of parameters in space-time, long or short time-scale, or small or large space-scale, the fractal dimension D_f appears “stably”, in a wide range of the parameter space.

What we have claimed in our papers is this “stability” of fractal dimension occurs in a certain wide range of parameters.

We have found a number of facts on the stability of fractal dimensions.

Unfortunately, at our present ability of understanding, we can not prove in general why these parameter regions give the stability to the fractal dimension for the trajectory of fluid particles.

Nevertheless, we find any indication that the “stability of the fractal dimension is related to the L´evy flight”. Indeed, the condition on the parameters to realize the L´evy flight is similar to that used to show the existence of critical distance r_c in Eqs. (12) and (13). That is, from Eq.(7), a trajectory jumps from t to t + δt by...

$$\delta \boldsymbol{x}(t) = \boldsymbol{x}(t+\delta t) - \boldsymbol{x}(t) = \frac{d_H}{r^D(t)}\delta t\ \boldsymbol{\Theta},$$

where the average

$$\boldsymbol{\Theta} = \langle \hat{\boldsymbol{d}}(t) - D\hat{\boldsymbol{x}}(t)(\hat{\boldsymbol{x}}(t)\cdot\hat{\boldsymbol{d}}(t))\rangle$$

is the order of 1.

Suppose the L’evy flight is realized by a big jump from the neighbor of the dipole to the the neighbor of boundaries, we have approximately the [L’evy flight Condition (LfC)] as

$$\delta \boldsymbol{x}(t) = \frac{d_H}{(2L_f)^D}\delta t\ \boldsymbol{\Theta} \approx 2L_f, \text{ or } \ [\mathrm{LfC}] \equiv \frac{d_H\ \delta t}{(2L_f)^{(D+1)}} \approx 1.$$

This affords a rough understanding of the parameter regions which give a stable fractal dimension D_f . Since δt = 0.01, if we take the typical values of d_H = 0.1 and L_f = 0.1, then [LfC] = 0.62 for 3D and 1.2 for 2D.” Our parameter regions in Tables 1-4 and Figure 4 (to appear shortly) is a little wider than this typical choice satisfying [LfC] ≈ 1.

(Question)

4 - Please address all other questions and suggestions from the report.

Minor changes:
1) - Several notations are not defined, defined incorrectly, or defined too late in the text. Please add the proper definition in words in the corresponding places:

[Reply on minor changes]
As explained below, we have clarified the various notations.

(Question)
- After Eq. (1) please replace "is the location of the source" by "are the locations of the sources".

(Answer)
We replace it.

(Question)
- In the footnote 1: please define SLE (used in [23]).

(Answer)
The word SLE in the footnote 1 has been explained:
"1 There is also literature discussing the relationship between turbulence and SLE(Schramm (or Stochastic) L¨oewner Evolution) using random fields [20, 21, 22], and it is expected that there may be a correspondence between these and our model."

(Question)
- After Eq. (3) please define the relations between ζi at different time steps: are they considered to be completely uncorrelated and random?
If yes, why this choice is physically relevant?

(Answer)
Yes, $\zeta_i(t)$ at different time steps are completely uncorrelated and random. This choice is physically acceptable; the random modulation of the velocity field of the fluid occurs independently in time and
in space. The reason we are taking the random modulation independent of time and space is that it is the simplest model.

(Question)
- dh is not defined in Eq. (6), only in the discussion after Eq. (7).

(Answer)
d_H has been more clearly defined by adding a sentence and modifying (6).
Before the sentence “We focus on a ...” in the above of Eq. (4), we add the following sentence:
“As was discussed in the above, the real turbulent phenomena is a very complicated one, which involves many vortices (eddies) of different sizes and vorticity, or many sources and sinks with different quantities of fluid (charge) Q coming in and out per unit time.
Here, we consider a simple toy model, in which there exists a single dipole (with a single

source and sink). More explicitly, the locations of the sink and source are identified, having a constant dipole moment, dH, but we keep the essential ingredient of random modulation which can be stated in other words, the magnitude of the dipole moment d_H = |**d**_H| is fixed time-independently, while the direction of the moment, **d**ˆ_H(t) = **d**_H(t)/d_H = is randomly (stochastically) modulated.
That is, we focus on ...”

(Question)
- double usage of the notation "n" in Eq. (1) for the number of sources and in Eq. (8) for the number of boxes occupied by the particle trajectory.

(Answer)
Usage of the same n has been resolved.

(Question)
- Subsection "Missing particles" in page 8: the notion of "the number of particles" is not defined. Eqs. (1-8) are written for the only particle:
please clarify the definition of the number of particles (probably with the number of numerical realizations).

(Answer)
We have responded in reply number Reply Ⅷ).

(Question)
- In the same subsection and page the scalar x_0 of the initial condition in the D-dimensional space is vaguely defined.
What does this scalar mean as a coordinate in the D-dimensional space?

(Answer)
It is spatially symmetric and therefore expressed as a scalar.
We have given the initial values in the actual calculation in a three-dimensional vector, taking only its x_0 component as nonzero.

(Question)
- Eq. (14) in page 11: ω and Γare not defined, as well as the subscripts 1,2,3.

(Answer)
The vortex ω and the circulation Γ have been clearly defined in Eqs. (16) and (17) and the sentence before them.

(Question)
- Double usage of the notation "N" in Sec. 3 as a number of time steps and after (17) as a number of dipoles.

(Answer)
The double definition of N is fixed.
Subscript i specifies each point-like vortex in 2D or string-like vortex filament in 3D, since we are considering a number of vortices here. In 2D case, as you pointed out, it is obscure, so we replace $X_{1,2}(t)$ by $X_{1,2}(t;i)$ in the first equation in (18).
In addition, the 3D case in equation (18) is also modified.

(Question)
- The abbreviation "N-S" after Eq. (20) is not defined.

(Answer)
No problem, the abbreviation of N-S was defined when Navier-Stokes equation appeared.

(Question)
- "The energy dissipation rate per unit mass" in Sec. 6.3 is not properly defined. It is not clear: "per unit mass" of what?

(Answer)
The word “energy dissipation rate per unit mass” is popular in fluid dynamics, but we add an explanation so as not to cause difficulty to the readers. $1/2\rho v^2$ is a kinetic energy per unit volume for a fluid or for others, if ρ is a density. Then, $1/2v^2$ is a kinetic energy per unit mass. The energy dissipation rate per unit mass, ε, can be defined as a time derivative of this, $\varepsilon = -d/dt(1/2v^2)$.
Put a footnote to the last sentence of the first paragraph in Sec. 6.3. That is, “Here, we examine the energy dissipation rate per unit mass ε_r.”
(Note: the difference of dissipation rate of energy density and energy dissipation rate per unit mass. The former is a complicated $-d/dt(1/2\rho v^2)_r$, but the latter is a simpler $-d/dt\,(1/2v^2)_r$. The subscript r specifies the quantity at a length scale r.)

(Question)
Eqs. (1-7) are written for the only particle.

(Answer)
The reviewer's understanding is correct, this is about a single particle. Missing Particle's discussion was about the statistics of that particle, and we explained that point in Reply Ⅷ).

(Question)
2) - Please add axis labels to figures 1 and 2.

(Answer)
We specify in the caption that it is on three-dimensional coordinates.

(Question)

3) - Please add proper descriptive captions to all figures in order to avoid searching through the text of their discussion.

(Answer)
We have added the setting values to Figures 3--7 as appropriate captions.

(Question)
4) - Please make figures in vector-graphic format to avoid degradation of its quality in a raster format.

(Answer)
We replaced the images with clearer ones.

(Question)
5) - The entire section 6 "Discussion" is put after the summary and all the results, therefore its relevance is doubtful.
If the authors consider this section to be important, they should move it to the relevant place of the manuscript.

(Answer)
We want to keep this style, the contents of the discussion are correct, and become useful in the near future.