Report

1. The authors investigated numerically anomalous diffusion in a randomly modulated dipolar velocity
field to construct a toy model for an anomalous diffusion of fluid particles in turbulence. The origin of the model is
explained in the discussion section 6. The authors calculated fractal dimensions of the trajectory in two and three dimensions
for two different types of boundary conditions including periodic ones and “Get back” to the original point conditions.

We appreciate for your correct understanding of our paper “Anomalous diffusion in a randomly modulated velocity Field”. We are happy to answer to your comments and revise our paper accordingly.

Requested changes

1. As I understand boundary conditions are needed because of the singularity in the dipolar field. Is this singularity, indeed, significant in liquids?
My first try for the problem like that would be to cut off the field divergence at certain radius $r=r_0$ for instance, replacing r with $r+r_0$ in the denominator in Eq. 7.
Then singularity will disappear. In that case the boundary conditions are not needed, and I would expect the fractal dimension to be equal to the space dimension.
Can the authors comment on that and include this case into the consideration?

Your first try of modifying the singular denominator in the velocity field by $r \rightarrow r + r_0$, is a good way to clarify what happens for a (fluid) particle in the neighborfood of the dipole. The revision is done just before the summary, with the new simulation following your suggestion. The result is that only in 3D with the boundary Condition 2, we have a modification, but in other cases, the new simulation does not differ so much from the original simulation without $r_0 = \Delta r$.

2. I guess the word "of" is missed in the third line of the next to the last paragraph in Sec. I.

Yes, you are right. Two [of]s are missing. We modify the sentence in the modification of before the 2nd paragraph from the bottom in Sec.1.

3. In the last part of eq. 5 should it be vector in the right hand side.

Something is wrong. Correctly, Eq.(5) after “such that” reads

$$\boldsymbol{d}_H(t) = \frac{2Q}{2\pi}\boldsymbol{\zeta}(t) = d_H \times \hat{\boldsymbol{d}}_H(t) \text{ finite.}$$

We have modified the explanation of the “dipole moment”.

4,. Table 3 contains zeros for dimensions in its second column. Why? Sorry if I missed explanations.

Yes, this Table 3 having 0.000 for d_H = 0.3 and 0.4 is correct. We have to explain it more definitely. This is related to the last sentence of Section 3. The sentence after “As detailed in the next section, ...” is not appropriate. It should be modified to “As detailed in the next section, the fractal dimension behaves in a similar pattern in 2D and 3D cases, and the degree of decrease for D_f depends only on dH. Especially, in Table 3, we find D_f = 0.000 for d_H = 0.3 and 0.4. This shows a phase transition-like effect exists for D_f by changing d_H. (See Eqs. (12) and (13)) The decrease of the particle number also occurs, depending on d_H.”

5. In addition to the velocity field there can exist the ordinary brownian motion contribution to the liquid particle dynamics. Is this right to ignore it?

Yes, you are right. There is always the ordinary Brownian motion, or the “weak” fluctuations for the fluid particle motion. It is however, the “strong ” (in other word singular) effects such as by the dipole modulation is necessary to induce the fractal behavior of the fluid trajectories. The fractal behavior of the fluid motions are required for the turbulence to occur. So, we take into account the effect which we consider being important.

6. Is there some way to verify the theory experimentally? Can the authors comment on that?

Very good question, but it is too early for us to comment on the experimental verification of our model at the level of a toy model. There are a number of computer simulations, but there are very few real experiments of observing the fractal nature of trajectories of a fluid particle or of a vortex. In reality, to trace the trajectory each by each is a rather difficult task, since we have to stain the fluid particle to see its trajectory, by adding a fluorescent material to a jet of a fluid fired into the other fluid. The problem is how to prepare a pair of sink and source. Please postpone our answer to this question.