

Summary of changes

In the version 2 of the manuscript, we show more numerical results to support the claim that the operator localizes in the Krylov basis in the *finite-size* MBL systems when initialized on the first site. On the other hand, since extrapolating the behavior of MBL systems from finite sizes to the thermodynamic limit is intricate and challenging, we modify our assertion regarding the linear extrapolation of the Lanczos coefficients. In particular, we keep the results of the linear extrapolation in the manuscript, but pivot it as the simplest first-step attempt to extrapolate the MBL Lanczos coefficients to the thermodynamic limit with cautions. We also adjust the format and the layout of the figures to match the journal style of SciPost.

Below, we list the changes and provide a table of changed figures for easy reference. We also include a copy of our manuscript with the modified parts colored as [blue](#).

1. In the Introduction (third last paragraph), we mention and discuss the recently raised skepticism regarding the existence of MBL in the thermodynamic limit, and citing various relevant references as requested by Referee. In this regard, we have carefully changed all the terms of “MBL phase” to “MBL systems” or “MBL regimes”. We also try to establish and provide more numerical results regarding the operator localization in the *finite-size* MBL systems. On the other hand, we pivot our result of the linear-extrapolation of the Lanczos coefficients as a simplest attempt to extrapolate the MBL system to the thermodynamic limit with cautions.
2. In Sec. 3.1, last paragraph, we add the discussion of the even-odd alteration. In particular, as requested by Referee, we stress that such an alteration has been observed in various systems. We also point out that the even-odd alteration is hidden under the an apparent randomness, which is a new feature of our result.
3. We separate the part of the linear-extrapolation of the Lanczos coefficients into Sec. 3.2. We add the first paragraph in Sec. 3.2 to reiterate again that the linear extrapolation is a simplest attempt to extract the thermodynamic-limit behavior, and a more sophisticated procedures might be needed.
4. In Sec. 3.3, we add:
 - Cumulative spectral functions for the systems in the ergodic and MBL regimes for several system sizes (Figs.4 (a)(b)).
 - The finite- n_{max} and finite L scaling of the amplitudes of the zero-frequency delta function (Figs.5 (a)(b)).
 - Spectral functions on the low-frequency part (Fig.6).

And the relevant discussions in the main text in Sec. 3.3. When applicable, we show the results with several system sizes as requested by Referee.

5. Add Sec. 3.4 and Fig. 7, which discuss the result of the zero mode, motivated by the suggestions and comments from Referee 2.
6. In Sec. 3.5, we add Fig. 8(a) and the second paragraph to show and discuss the Krylov complexity for several finite sizes in the ergodic and MBL systems.
7. In Sec. 3.5, we add Fig. 9 and the fourth paragraph to show and discuss the wavefunction profile at an exponentially long time and the long-time averaged wavefunction profile.

8. In Sec. 3.5, we add Fig. 11 and the last paragraph to show and discuss the finite- n_{max} and finite- L scaling of the return probability, to further support the claim of the operator localization.
9. In Sec. 4, at various places in the main text, we add discussions regarding the reason of the apparent different behavior of the Lanczos coefficients in the MBL phenomenological model. We also discuss some similarity of the calculation to the calculation of the single-spin dynamics.
10. We modify the section Conclusion accordingly.
11. We add additional data of the Lanczos coefficients in Appendix B and Figs.15-18.

Figure Changes		
Version 2	Version 1	Notes
Figure 1 (a)	Figure 1	
Figure 1 (b)	Figure 2	Merges Fig.1 & Fig.2
Figure 2	Figure 3	Includes more disorder realizations
Figure 3	Figure 4	
Figure 4 (a)(b)	New	
Figure 4 (c)	Fig. 5	
Figure 5	New	
Figure 6	New	
Figure 7	New	
Figure 8(a)	New	
Figure 8(b)	Fig. 6	
Figure 9	New	
Figure 10(a)	New	Inset from v1 Fig.7(a)
Figure 10(b)	Fig.7 (b)	
Figure 11	New	
Figure 12	Figure 8	system size changed to $L = 14$
Figure 13 (a)	Figure 9	3
Figure 13 (b)	Figure 10	system size changed to $L = 14$