

Response to Referee 1

We thank referee for his/her concise summary and high assessments of the importance of our work, and the useful comments and suggestions. We are more than happy to implement the requested changes and have substantially revised the main text and SM accordingly. Below are our responses to the requests:

Requested changes 1: All the SM should be turned into regular appendices or integrated into the main text.

Reply 1: Thanks for the suggestion and we have turned the SM into regular appendices.

Requested changes 2: The authors should provide details regarding the quality of the fits of their data to Eq. (4) in the form of χ^2 values that add information beyond the statement in [49].

Reply 2: Thanks for the suggestion and we have added a more careful analysis of the fits in the Fig. 4 in the revised Appendix A.

Requested changes 3: The definition of the perimeter $l = 4R - R$ should be justified (why not $l = 4R$)?

Reply 3: Sorry for the confusion. Since l is the perimeter of region M and the M is a square shaped area with linear length R , we use $l = 4R - 4$ to subtract the overcounting of the 4 corners of the M . Of course, such a subtraction will not affect the scaling behavior in the thermodynamic limit.

Comment 4: The authors should explain how they arrived at the estimated scaling dimension for the spin operator in [53]. Furthermore, it should be made more explicit, how the last sentence in [53] relates to that result, i.e., the smallness of the imaginary part (I suppose).

Reply 4: The scaling dimension is extracted from Ref. [52] ([55] in the revised manuscript). We have also expanded the discussions about the disorder operator in 1+1d Potts model.

Comment 5: In the current SM subsection on the choice of M , the authors claim that for the Even-A subregion the negative corner contribution comes about because of a large perimeter contribution that affects the fitting prediction. If this was indeed the case, then why does a similar problem not affect/explain the observed negative value of s in the case of the J-Q3 model? Would the large-angle values of s turn out to become positive for the J-Q3 model if better data would be available?

Reply 5: Thanks for the question. Actually, as shown in Fig.9 of the revised manuscript (in the appendix), the Even-A indeed affect the scaling behavior, note that even in Fig.9 is the Even-A in the Fig.7 for the J1-J2 model. That is why, we have to introduce the even- \tilde{B} approach for the J-Q3. And as for the large angle value of s , as shown in the Fig.9 (c), both even (the Even-A) and even- \tilde{B} give negative values, in fact the even (the Even-A) is more negative than even- \tilde{B} . So we do not think s will turn to positive for J-Q3 at large angle.

Comment 6:- 6. The authors should show (e.g. included in the current Fig. S5) the results of a naive measurement of $\langle X_M \rangle$ at the anticipated DQC point in order for the reader to compare to the results from their correction procedure.

Reply 6: Thanks for the suggestion. In the revised Fig.10 (in the appendix), we have now included the $s(\pi)$ as a function of $1/L$ for $q_c = 0.59864$ at the DQCP.

Comment 7:- 7. Overall, the manuscript contains a large number of language flaws, which must be corrected. Below is an incomplete list focusing on the first page only:

-title: “Scaling of disorder operator...” – > “Scaling of the disorder operator...”

-page 1: “exciciting” – > “exciting”

-page 1: “which is defined” – > “which are defined”

-page 1: “U(1) disorder operator” – > “The U(1) ...”

-page 1: “one side of DQC exhibits the Valence Bond Solid” – > “one site of the DQC exhibits valence bond solid”

-page 1: “behavior of disorder operator” – > “behavior of the disorder operator”

-page 1: “on square lattice, using the ubiased Stochastic...” – > “on the square lattice, using unbiased stochastic ...”

Reply 7: We thank the referee for the careful reading of our manuscript. We have tried our best to correct these language flaws, such as inserting all the missing *the*, hopefully the text are better now, we have also corrected few typos.