Dear Editor,

We would like to resubmit our paper entitled "Thermal and dissipative effects on the heating transition in a driven critical system" for publication in SciPost Physics. We thank the referees for their elaborate comments and suggestions. Many of these comments triggered improvements in our manuscript. We would particularly like to draw your attention to the addition of new theoretical CFT results (Sec. 4.1) to the manuscript which enriches the analysis of the results obtained directly on the discrete lattice system. We have added new figures which illustrate in a clearer fashion both the impact of finite size effects and increased dissipation.

Based on the criteria of SciPost Physics, our article is in sync with both criteria 3 and 4: our work establishes a synergy between CFTs, quantum thermodynamics, open systems in a concrete and computable model. These connections are particularly useful, as they provide answers to highly complex and often intractable questions concerning asymptotic time behaviours and issues of thermalization which are of intense interest to the condensed matter and quantum engineering communities. Secondly, our observation that CFTs still provide a promising framework for analysing driven-dissipative systems motivates the further exploration of non-unitary CFTs in this context.

Given the positive appraisal of Referee 2 who recommended publication and the inclusion of new results (both theoretical and numerical) in the current version, we feel that our paper warrants publication as a regular article in SciPost Physics.

Sincerely,
The authors

1 Reply to Referee 1 :

The findings presented in this work definitely warrant publication, but it is not entirely clear to me that it meets the stringent acceptance criteria for SciPost Physics. From a methodological point of view the paper is in my view very straightforward, while on the other hand no unexpected new phenomena are observed. However, I am sure the authors would argue that this absence is in itself a key result of the work (and I would agree with this).

We thank Referee 1 for their appreciation of our work. However, we would like to highlight the following points that in our opinion make our manuscript fulfil acceptance criteria for SciPost Physics.

In our previous works, we showed that the heating phase diagram predictions of the CFT were universal for any driven critical theory (irrespective of whether the underlying physical lattice models are interacting or not). Quantities like energy and entropies were on the other hand determined exclusively by the central charge of the theory. In our current work, our aim was to understand the interplay between drive and dissipation, especially in the context of the emergent horizons in the closed system. From this perspective, we focused on the simple free lattice fermions, as this reunites both the nontrivial physics of heating under drive and exact solvability in the presence of dissipation. In our view, this model captures the essential interplay between drive, thermal effects and dissipation.

To bolster this viewpoint, we have added a new subsection 4.1, where we provide an exact analytical result for the energy growth $E(t)$ from CFT starting from any initial temperature β^{-1}. This result is new in the Floquet CFT literature and gives an analytical proof of the resilience of the pure state results to thermal initial states. On the other hand, it enriches our lattice versus CFT comparison to better disentangle CFT and pure lattice effects. In particular, we observe new effects intrinsic to the lattice, such as damping of the energy in the non-heating phase. This is shown in the updated Figs. 2 (c,d).

While the dissipative effects cannot be captured by our CFT calculations, we find, remarkably, that the entanglement properties of the horizons x_* and $L - x_*$ still survive the addition of dissipation and dephasing to a certain extent. Despite specific lattice effects in the open system, universal properties such as the Floquet CFT phase transition persist for short to intermediate times. Furthermore, we stress that the strong resilience (even to large values of γ) of the horizons in the high-frequency regime to dissipation (see Fig. 6) is a remarkable result that shows explicitly that no information can flow from the left to the right dissipative site. We believe this result is in itself unexpected and differs with a key feature of the horizon physics of the heating phase of the Floquet CFT: the horizons here act as a blockade of energy and particles, not as energy hotspots. We stressed this point further in the new version of the manuscript.

1. In Fig. 3 ten cycles are considered. An obvious question is how these pictures change as the number of cycles is increased.

We thank the referee for this comment. In Fig. 1 we show that the phase diagram remains qualitatively the same even for a larger number cycles, in particular for 20 Floquet cycles instead of 10. However, we stress that the regime of validity of the CFT predictions in the heating phase is typically 10 cycles (or less if the initial temperature is high), therefore although energy absorption is still faster in the heating than in the non-heating regime after 20 Floquet cycles, the behaviour of the total energy $E(t)$ will not follow CFT predictions anymore. We have added a short explanation in the caption of Fig. 3 emphasising this.

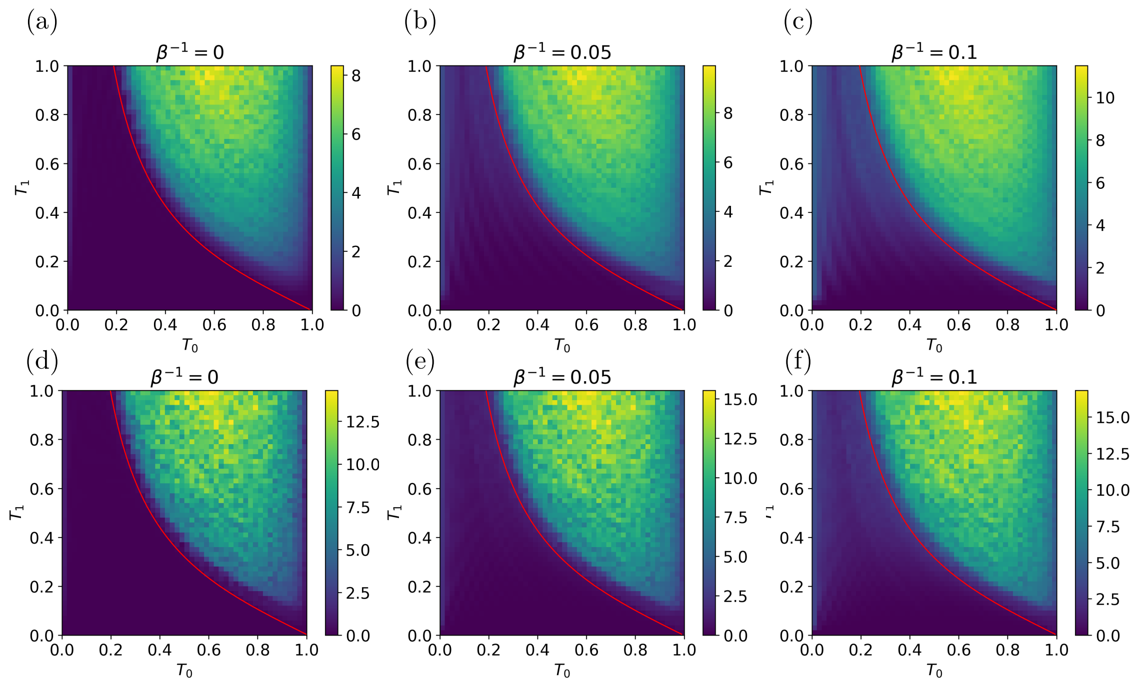

Figure 1: Total energy after 10 Floquet cycles as a function of T_0 and T_1 for $L = 200$, for (a) $\beta^{-1} = 0$, (b) $\beta^{-1} = 0.05$, (c) $\beta^{-1} = 0.1$. (d-f): Same, but for 20 Floquet cycles

2. In Fig. 2 much lower temperatures are considered than in Fig.2. This is unfortunate in the sense that Fig. 3 (b) and (c) cannot be related to any curves in Fig. (2).

We thank the referee for their remark. We changed the temperatures displayed on Fig. 3. In particular, we took the initial temperatures $\beta^{-1} = 0$, $\beta^{-1} = 0.05$ and $\beta^{-1} = 0.1$, such that $\beta^{-1} = 0.05$ can be compared explicitly with the new version of Figure 2. For $\beta^{-1} = 0.1$, however, we did not display the growth of energy $E(t)$ as in this case the oscillatory behaviour decays very rapidly, and the expected behaviour from non-heating phase does not really hold anymore. Nonetheless, a much faster energy absorption is seen in the heating regime, which still leads to the expected phase diagram.

3. The dissipative with particle loss/gain is special because the density matrix remains Gaussian (i.e. the full model is integrable). This ceases to be the case for the dephasing noise. A question I have is whether there are any interesting effects if both types of dissipative couplings are present, i.e. particle/gain loss and dephasing on top (making the model non-integrable).

The referee's suggestion is indeed very interesting. In systems without any Floquet driving, for example the XXZ chain, it has been shown that the interplay between edge dissipators and local dephasing terms can indeed result in varied regimes of heat and spin transport, for instance unidirectional heat flow, or heat flowing from both edge reservoirs towards the middle. Transitions from a ballistic regime to diffusive regime can be generated by such dissipators [1]. When coupled with periodic driving, it is highly likely that interesting regimes of behaviour, especially from a quantum thermodynamics perspective, might arise. However, answering this question requires extensive numerical calculations, e.g., using the TEBD algorithm, which are beyond the scope of the current work. We have nonetheless added a comment about this in our paper now.

2 Reply to Referee 2 :

We thank referee 2 for their appreciation of our work, as well as their interesting suggestions to significantly improve our manuscript. Below are our responses to the points they raised.

Fig. 5 and Fig. 7 show the mutual information as a function of the subsystem site (if I understand property). Since the claim the author make is about a linear growth in the stroboscopic time (within the central region), it would be useful to see the plot of mutual information vs N_{cycles}, to check that such linear growth

We thank the referee for their remark. For Fig. 5, we added a plot in both figures showing the linear growth of the half-system mutual information, as predicted from CFT, for two parameter regimes: $T_0/L = 0.95$, $T_1/L = 0.05$, away from the high-frequency regime, as well as $|T_0/L| = T_1/L = 0.05$, in the high-frequency regime of the heating phase, see Fig. 2(a). Both parameter regimes have the same heating rate, and thus the same linear growth of half-system mutual information. However, we observe a higher robustness of the linear growth to initial temperatures for the high-frequency regime, while the micromotion and the open boundary conditions affect such growth in the low-frequency regime, where $T_0 + T_1 \sim L$.
For Fig. 7, we carried out the same analysis in the dissipative case, see Fig. 2(b): in the high-frequency regime of the heating phase, the linear growth of mutual information is robust to values of γ higher than 0.005, while in the low-frequency regime $T_0 + T_1 \sim L$, the linear growth is still observed at higher values of γ but the heating is significantly lowered as a consequence of micromotion.

To summarize, we would like to emphasize that there are several regions in the phase diagram, where reduced micromotion makes it possible to clearly observe the linear growth of mutual information for a few tens of cycles. We added this in a figure to the manuscript, as the concluding Figure 11.

- *The authors comment about energy accumulation at x_* and $(L - x_*)$ being visible in Fig. 6b. However, it does not seem to me that a clear pick is there, but rather a whole region extending from the edges to those points. Maybe they could show more values of gamma and/or comment more about that.*

We thank the referee for their remark, as this point was not made clear in the previous version of the manuscript. Energy and particle density accumulate at any point in the intervals $[0, x_*]$ and $[L - x_*, L]$ as a direct effect of the particle exchanges taking place at site 0 and site L. However in the high-frequency regime, the horizon physics of the heating phase is robust and energy/particles cannot flow through the horizons, leading to a purely constant energy and particle density in the interval $[x_*, L - x_*]$. In this sense, the horizons can be seen very clearly as the two points that separate a region with constant energy and particle densities and regions of growing energy and particle densities. We made this point clearer in the manuscript, and we highlighted this non-trivial result about energy blockage in the conclusion of our manuscript.

- *The authors claim the CFT prediction being robust to dissipation for values of $\gamma < 0.005$. This value is however very small. I wonder whether this means that this robustness only holds in a certain scaling limit. To this aim it would have seem natural to me to investigate this gamma-range as a function of the system/subsystem size.*

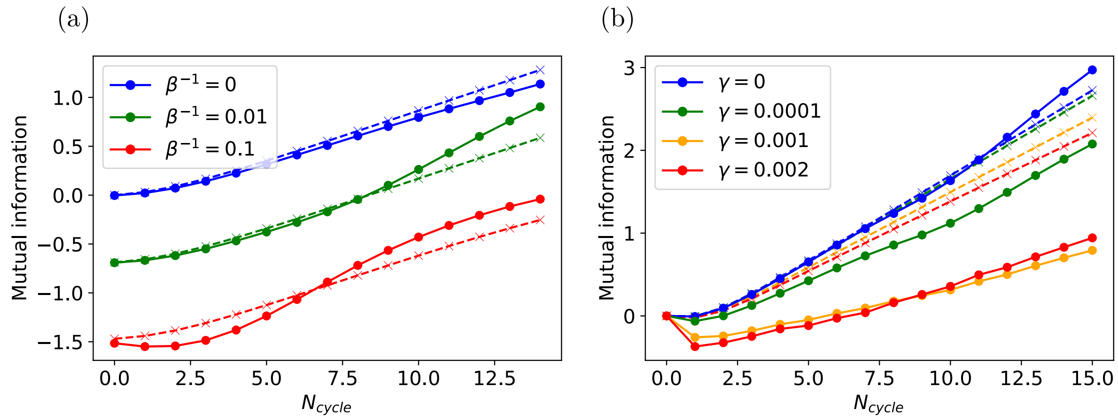

Figure 2: (a) Growth of half-system mutual information $I([0, L/2], [L/2, L])$ for $T_0/L = 0.95$, $T_1/L = 0.05$ (full lines), $T_0/L = -0.05$, $T_1/L = 0.05$ (dashed lines), and different initial temperatures. The equilibrium value of half-system mutual information in the ground state has been subtracted. (b) Growth of half-system mutual information $I([0, L/2], [L/2, L])$, for $T_0/L = 0.9$, $T_1/L = 0.1$ (full lines), $T_0/L = -0.1$, $T_1/L = 0.1$ (dashed lines), and different dissipations γ.

We thank the referee for this important remark about the range of validity of the CFT predictions to robustness. We would like to stress that there are two distinct parameter regimes of the heating phase which can be discussed: the high-frequency regime and the low-frequency one. While we focused our analysis on the low-frequency one, which has very large micromotion, it is equally interesting to also analyse the high-frequency regime. For the high-frequency regime, the linear growth in mutual information deviates only slightly from the dissipationless CFT predictions for range of γ upto $\gamma \sim 0.005$ (see Fig. Fig. 3(a)). However the linear growth of mutual information persists for way larger values of dissipation, upto $\gamma \sim 0.05$, as system size is increased. To check this, we consider half-system mutual information, which should always grow linearly regardless of the choice of the driving parameters in the heating phase (as both subsystems A and B contain one horizon each), and consider for different values of γ different total system sizes L. The curves for different system sizes collapse to a single curve for low enough values of dissipation, while for higher values there is a faster deviation from the linear growth for smaller system sizes L. Furthermore, for $\gamma = 0.1$, mutual information is not growing linearly even for system size $L = 800$, as observed on Fig. 3(b) We added the following analysis and figure to the manuscript as we believe it makes the discussion of the validity of the linear growth against dissipation clearer.

- In the introduction: "Exceptions to this paradigm include some integrable disordered systems [30] as well as many-body localized systems [31] which evade heating." Maybe the authors meant "do not evade heating"? I think that at least Ref.30 is about some integrable disorder system is shown to heat up.

We thank the referee for their remark. There was a typo with a missing comma: "Exceptions to this paradigm include some integrable disordered systems [30] , as well as many-body localized systems [31] which evade heating.". The paradigm in question is the equilibration to a GGE, which Ref. [30] provides a counterexample in a heating integrable system, while MBL provides a counterexample in a system evading heating. We have corrected this misunderstanding in our paper.

- Capture Fig. 1: "We want to understand the robustness of this phenomenology to the introduction of dissipation". I would stress that also thermal effects are of interest here.

We now also refer to thermal effects here, and stressed the derivation of new Floquet CFT results at finite temperature and finite size.

- Section 2: "Using the tools developed in Ref. [47,48]". I think that the transformations in Eq. 2 were

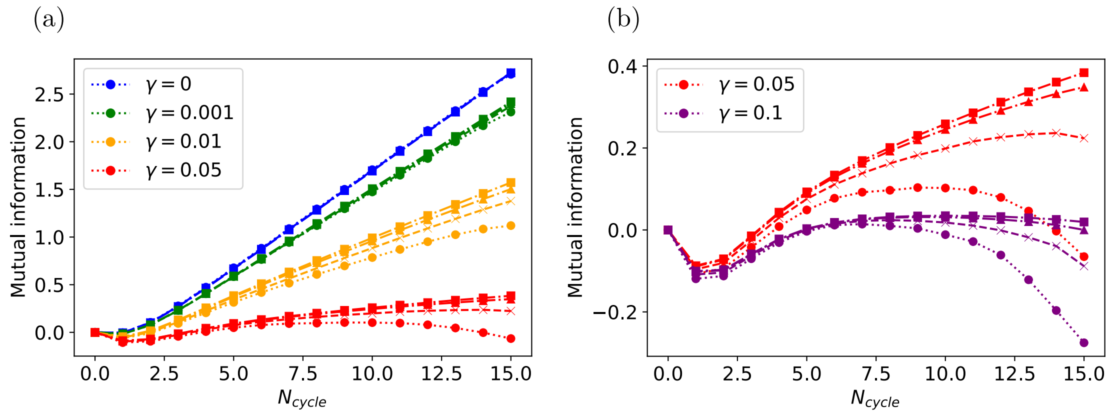

Figure 3: (a) Growth of half-system mutual information $I([0, L/2], [L/2, L])$ for $|T_0/L| = 0.05$, $T_1/L = 0.05$, with different dissipations, and system sizes $L = \{100, 200, 500, 800\}$ (circle, cross, triangle, square). (b) Same plot for different values of γ.

first given in this context in SciPost Phys. 2, 002 (2017) and SciPost Phys. 6, 051 (2019).

We thank the referee for pointing these references out. We added them to the manuscript.

- Section 4 (below eq. 27): $h_0 \to H_0$ *?*

We thank the referee for this remark, but we decided to keep h_0, in order to stress that H_0 is an operator in the second-quantized language while h_0 is a finite-dimensional matrix.

- Section 4: About the extension of the Floquet-CFT results to finite temperature the authors claim "This complexity can be traced to the fact that for thermal states, the correlation functions cannot be fixed solely by conformal invariance, but require the knowledge of the full operator content of the theory at hand ". I honestly do not understand this point as, to my knowledge, zero and finite temperature correlations are simply related by a conformal transformation. I am probably missing something, but I think the authors could try to explain this better.

We stress that the Floquet CFT phenomenology only appears at finite system size L. Usual conformal transformations relate the Euclidean spacetime cylinder at finite size and zero temperature to the thermal cylinder at finite temperature and infinite size, while we want to have both finite system size L and finite temperature β^{-1}, giving a Euclidean space-time torus which cannot be mapped to a cylinder via a conformal transformation. This requires the knowledge of the full operator content of the theory at hand, as explained in the manuscript. See Sec. 3.1.1 in [2] for a thorough discussion of the question of finite size *and* finite temperature. However, we removed this paragraph in the current version of the manuscript as we actually managed to derive the finite temperature time-evolution of the energy from CFT at finite size, using the torus partition function of the compactified free boson CFT, together with the 2×2 representations of the $\mathfrak{su}(1, 1)$ algebra and the Floquet Hamiltonian, see Sec. 4.1.

3 List of changes

In the revised version of the manuscript, the following was changed:

1. The abstract and introduction were partially rewritten to make explicit mention of the new results on the CFT side at finite initial temperature.

2. Caption of Fig. 1 was changed according to the comment from Referee 2.

3. The Refs. [47,48] were added to the manuscript, as suggested by Referee 2.
4. Section 4 was fully rewritten. In particular, we added Sec. 4.1 that deals with the analytic computation of the energy evolution starting from thermal state from CFT.
5. Figure 2 was changed to make direct comparison between CFT and lattice results at finite temperature. Plots showing long time evolution of the energy in the non-heating phase were also added in Fig. 2(c,d).
6. In Sec. 4.2, the analysis of the total energy growth was rewritten to discuss CFT comparison as well as energy damping effects in the non-heating phase.
7. Figure 3 was changed according to the comment of Referee 1: the initial temperatures now displayed are $\beta = \{0, 0.05, 0.1\}$, in order to have explicit comparison between Fig. 3(a,b) and Fig. 2.
8. Figure 7 was added as suggested by Referee 2, together with a paragraph on page 16.
9. A paragraph was added on page 18 about adding both dissipation and dephasing, as suggested by Referee 1
10. The conclusion was partially rewritten to take into account the new changes in both Sec. 4 and 5. In particular, Figure 11 was added, as suggested by Referee 2.

References

[1] J. J. Mendoza-Arenas, S. Al-Assam, S. R. Clark, and D. Jaksch, "Heat transport in the xxz spin chain: from ballistic to diffusive regimes and dephasing enhancement," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2013, no. 07, p. P07007, jul 2013. [Online]. Available: https://doi.org/10.1088%2F1742-5468%2F2013%2F07%2Fp07007

[2] P. Calabrese and J. Cardy, "Entanglement entropy and conformal field theory," *Journal of Physics A: Mathematical and Theoretical*, vol. 42, no. 50, p. 504005, dec 2009. [Online]. Available: https://doi.org/10.1088%2F1751-8113%2F42%2F50%2F504005