1 Reply to Referee 1 :

The findings presented in this work definitely warrant publication, but it is not entirely clear to me that it meets the stringent acceptance criteria for SciPost Physics. From a methodological point of view the paper is in my view very straightforward, while on the other hand no unexpected new phenomena are observed. However, I am sure the authors would argue that this absence is in itself a key result of the work (and I would agree with this).

We thank Referee 1 for their appreciation of our work. However, we would like to highlight the following points that in our opinion make our manuscript fulfil acceptance criteria for SciPost Physics.

In our previous works, we showed that the heating phase diagram predictions of the CFT were universal for any driven critical theory (irrespective of whether the underlying physical lattice models are interacting or not). Quantities like energy and entropies were on the other hand determined exclusively by the central charge of the theory. In our current work, our aim was to understand the interplay between drive and dissipation, especially in the context of the emergent horizons in the closed system. From this perspective, we focused on the simple free lattice fermions, as this reunites both the nontrivial physics of heating under drive and exact solvability in the presence of dissipation. In our view, this model captures the essential interplay between drive, thermal effects and dissipation.

To bolster this viewpoint, we have added a new subsection 4.1, where we provide an exact analytical result for the energy growth $E(t)$ from CFT starting from any initial temperature β^{-1}. This result is new in the Floquet CFT literature and gives an analytical proof of the resilience of the pure state results to thermal initial states. On the other hand, it enriches our lattice versus CFT comparison to better disentangle CFT and pure lattice effects. In particular, we observe new effects intrinsic to the lattice, such as damping of the energy in the non-heating phase. This is shown in the updated Figs. 2 (c,d).

While the dissipative effects cannot be captured by our CFT calculations, we find, remarkably, that the entanglement properties of the horizons x_* and $L - x_*$ still survive the addition of dissipation and dephasing to a certain extent. Despite specific lattice effects in the open system, universal properties such as the Floquet CFT phase transition persist for short to intermediate times. Furthermore, we stress that the strong resilience (even to large values of γ) of the horizons in the high-frequency regime to dissipation (see Fig. 6) is a remarkable result that shows explicitly that no information can flow from the left to the right dissipative site. We believe this result is in itself unexpected and differs with a key feature of the horizon physics of the heating phase of the Floquet CFT: the horizons here act as a blockade of energy and particles, not as energy hotspots. We stressed this point further in the new version of the manuscript.

1. In Fig. 3 ten cycles are considered. An obvious question is how these pictures change as the number of cycles is increased.

We thank the referee for this comment. In Fig. 1 we show that the phase diagram remains qualitatively the same even for a larger number cycles, in particular for 20 Floquet cycles instead of 10. However, we stress that the regime of validity of the CFT predictions in the heating phase is typically 10 cycles (or less if the initial temperature is high), therefore although energy absorption is still faster in the heating than in the non-heating regime after 20 Floquet cycles, the behaviour of the total energy $E(t)$ will not follow CFT predictions anymore. We have added a short explanation in the caption of Fig. 3 emphasising this.

2. In Fig. 2 much lower temperatures are considered than in Fig.2. This is unfortunate in the sense that Fig. 3 (b) and (c) cannot be related to any curves in Fig. (2).

We thank the referee for their remark. We changed the temperatures displayed on Fig. 3. In particular, we took the initial temperatures $\beta^{-1} = 0$, $\beta^{-1} = 0.05$ and $\beta^{-1} = 0.1$, such that $\beta^{-1} = 0.05$ can be compared explicitly with the new version of Figure 2. For $\beta^{-1} = 0.1$, however, we did not display the growth of energy $E(t)$ as in this case the oscillatory behaviour decays very rapidly, and the expected behaviour from non-heating phase does not really hold anymore. Nonetheless, a much faster energy absorption is seen in the heating regime, which still leads to the expected phase diagram.

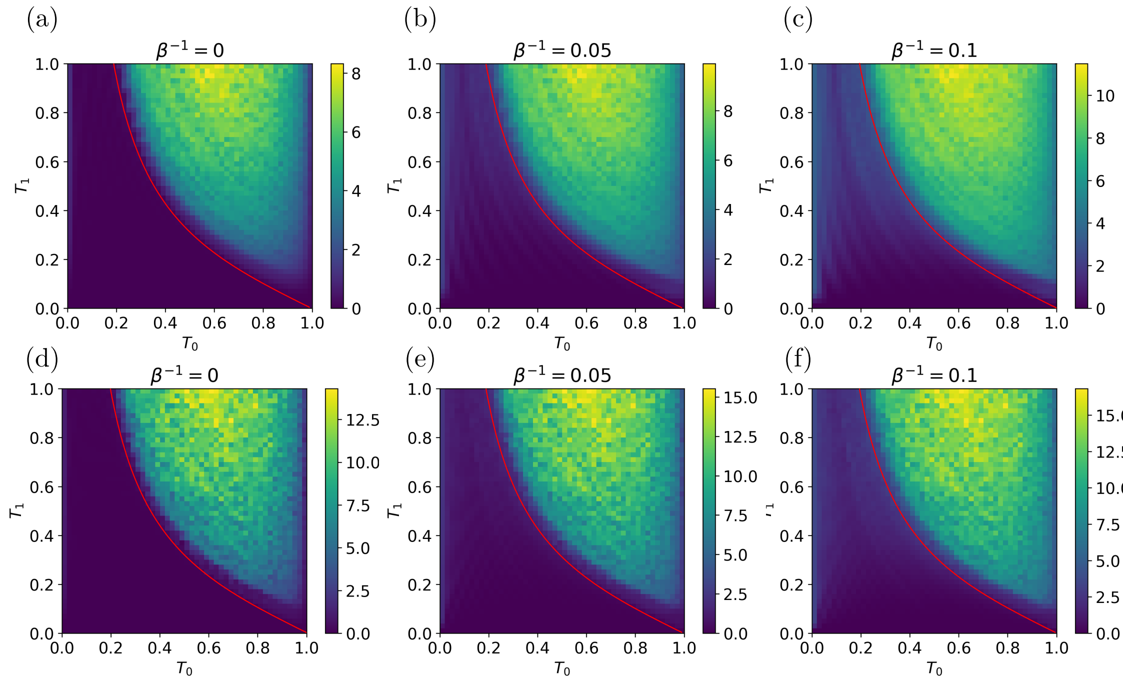

Figure 1: Total energy after 10 Floquet cycles as a function of T_0 and T_1 for $L = 200$, for (a) $\beta^{-1} = 0$, (b) $\beta^{-1} = 0.05$, (c) $\beta^{-1} = 0.1$. (d-f): Same, but for 20 Floquet cycles

3. The dissipative with particle loss/gain is special because the density matrix remains Gaussian (i.e. the full model is integrable). This ceases to be the case for the dephasing noise. A question I have is whether there are any interesting effects if both types of dissipative couplings are present, i.e. particle/gain loss and dephasing on top (making the model non-integrable).

The referee's suggestion is indeed very interesting. In systems without any Floquet driving, for example the XXZ chain, it has been shown that the interplay between edge dissipators and local dephasing terms can indeed result in varied regimes of heat and spin transport, for instance unidirectional heat flow, or heat flowing from both edge reservoirs towards the middle. Transitions from a ballistic regime to diffusive regime can be generated by such dissipators [1]. When coupled with periodic driving, it is highly likely that interesting regimes of behaviour, especially from a quantum thermodynamics perspective, might arise. However, answering this question requires extensive numerical calculations, e.g., using the TEBD algorithm, which are beyond the scope of the current work. We have nonetheless added a comment about this in our paper now.

References

[1] J. J. Mendoza-Arenas, S. Al-Assam, S. R. Clark, and D. Jaksch, "Heat transport in the xxz spin chain: from ballistic to diffusive regimes and dephasing enhancement," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2013, no. 07, p. P07007, jul 2013. [Online]. Available: https://doi.org/10.1088%2F1742-5468%2F2013%2F07%2Fp07007