1 Reply to Referee 2 :

We thank referee 2 for their appreciation of our work, as well as their interesting suggestions to significantly improve our manuscript. Below are our responses to the points they raised.

Fig. 5 and Fig. 7 show the mutual information as a function of the subsystem site (if I understand property). Since the claim the author make is about a linear growth in the stroboscopic time (within the central region), it would be useful to see the plot of mutual information vs N_{cycles}*, to check that such linear growth*

We thank the referee for their remark. For Fig. 5, we added a plot in both figures showing the linear growth of the half-system mutual information, as predicted from CFT, for two parameter regimes: $T_0/L = 0.95$, $T_1/L = 0.05$, away from the high-frequency regime, as well as $|T_0/L| = T_1/L = 0.05$, in the high-frequency regime of the heating phase, see Fig. 1(a). Both parameter regimes have the same heating rate, and thus the same linear growth of half-system mutual information. However, we observe a higher robustness of the linear growth to initial temperatures for the high-frequency regime, while the micromotion and the open boundary conditions affect such growth in the low-frequency regime, where $T_0 + T_1 \sim L$.
For Fig. 7, we carried out the same analysis in the dissipative case, see Fig. 1(b): in the high-frequency regime of the heating phase, the linear growth of mutual information is robust to values of γ higher than 0.005, while in the low-frequency regime $T_0 + T_1 \sim L$, the linear growth is still observed at higher values of γ but the heating is significantly lowered as a consequence of micromotion.

To summarize, we would like to emphasize that there are several regions in the phase diagram, where reduced micromotion makes it possible to clearly observe the linear growth of mutual information for a few tens of cycles. We added this in a figure to the manuscript, as the concluding Figure 11.

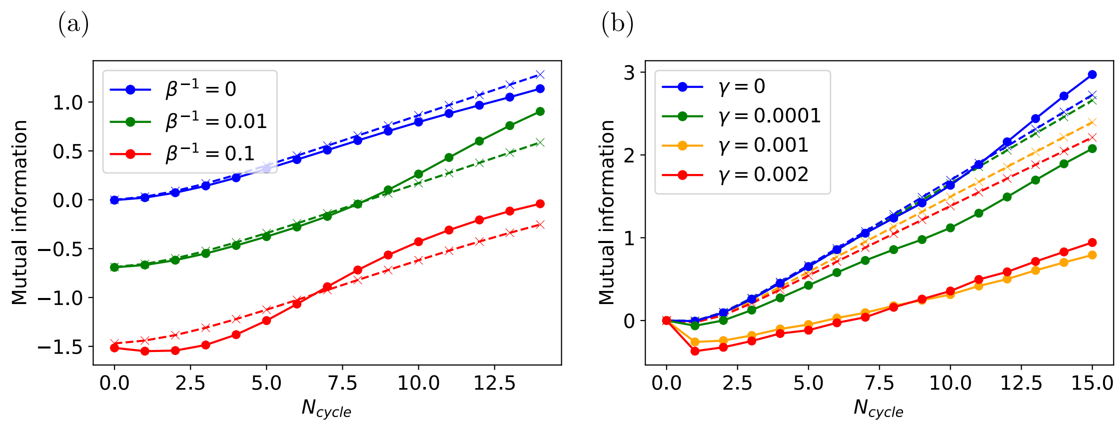

Figure 1: (a) Growth of half-system mutual information $I([0, L/2], [L/2, L])$ for $T_0/L = 0.95$, $T_1/L = 0.05$ (full lines), $T_0/L = -0.05$, $T_1/L = 0.05$ (dashed lines), and different initial temperatures. The equilibrium value of half-system mutual information in the ground state has been subtracted. (b) Growth of half-system mutual information $I([0, L/2], [L/2, L])$, for $T_0/L = 0.9$, $T_1/L = 0.1$ (full lines), $T_0/L = -0.1$, $T_1/L = 0.1$ (dashed lines), and different dissipations γ.

- The authors comment about energy accumulation at x_* *and* $(L - x_*)$ *being visible in Fig. 6b. However, it does not seem to me that a clear pick is there, but rather a whole region extending from the edges to those points. Maybe they could show more values of gamma and/or comment more about that.*

We thank the referee for their remark, as this point was not made clear in the previous version of the manuscript. Energy and particle density accumulate at any point in the intervals $[0, x_*]$ and $[L - x_*, L]$ as a direct effect of the particle exchanges taking place at site 0 and site L. However in the high-frequency regime, the horizon physics of the heating phase is robust and energy/particles cannot flow through the horizons, leading to a purely constant energy and particle density in the interval $[x_*, L - x_*]$. In this sense, the horizons can be seen very clearly as the two points that separate a region with constant energy and particle densities and regions of growing energy and particle densities. We made this point clearer in the manuscript, and we highlighted this non-trivial result about energy blockage in the conclusion of our manuscript.

- The authors claim the CFT prediction being robust to dissipation for values of $\gamma < 0.005$. This value is however very small. I wonder whether this means that this robustness only holds in a certain scaling limit. To this aim it would have seem natural to me to investigate this gamma-range as a function of the system/subsystem size.

We thank the referee for this important remark about the range of validity of the CFT predictions to robustness. We would like to stress that there are two distinct parameter regimes of the heating phase which can be discussed: the high-frequency regime and the low-frequency one. While we focused our analysis on the low-frequency one, which has very large micromotion, it is equally interesting to also analyse the high-frequency regime. For the high-frequency regime, the linear growth in mutual information deviates only slightly from the dissipationless CFT predictions for range of γ upto $\gamma \sim 0.005$ (see Fig. Fig. 2(a)). However the linear growth of mutual information persists for way larger values of dissipation, upto $\gamma \sim 0.05$, as system size is increased. To check this, we consider half-system mutual information, which should always grow linearly regardless of the choice of the driving parameters in the heating phase (as both subsystems A and B contain one horizon each), and consider for different values of γ different total system sizes L. The curves for different system sizes collapse to a single curve for low enough values of dissipation, while for higher values there is a faster deviation from the linear growth for smaller system sizes L. Furthermore, for $\gamma = 0.1$, mutual information is not growing linearly even for system size $L = 800$, as observed on Fig. 2(b) We added the following analysis and figure to the manuscript as we believe it makes the discussion of the validity of the linear growth against dissipation clearer.

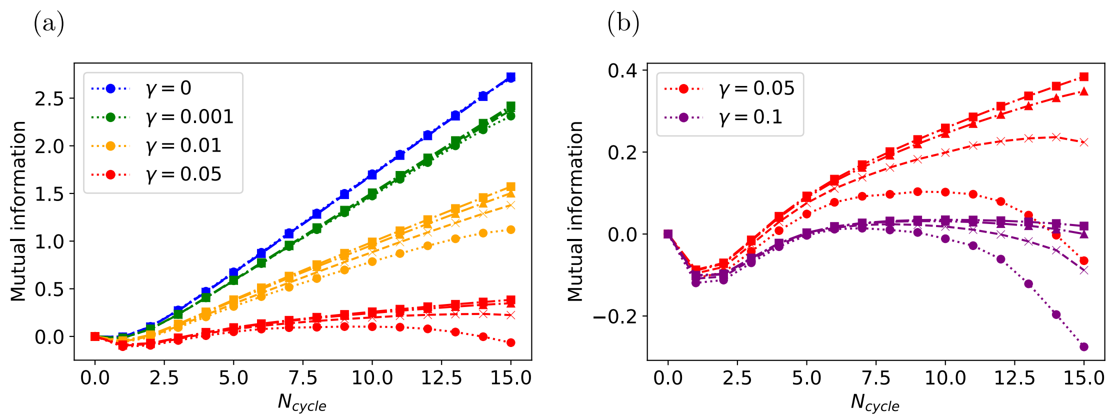

Figure 2: (a) Growth of half-system mutual information $I([0, L/2], [L/2, L])$ for $|T_0/L| = 0.05$, $T_1/L = 0.05$, with different dissipations, and system sizes $L = \{100, 200, 500, 800\}$ (circle, cross, triangle, square). (b) Same plot for different values of γ.

- In the introduction: "Exceptions to this paradigm include some integrable disordered systems [30] as well as many-body localized systems [31] which evade heating." Maybe the authors meant "do not evade heating"? I think that at least Ref.30 is about some integrable disorder system is shown to heat up.

We thank the referee for their remark. There was a typo with a missing comma: "Exceptions to this paradigm include some integrable disordered systems [30] , as well as many-body localized systems [31] which evade heating.". The paradigm in question is the equilibration to a GGE, which Ref. [30] provides a counterexample in a heating integrable system, while MBL provides a counterexample in a system evading heating. We have corrected this misunderstanding in our paper.

- Capture Fig. 1: "We want to understand the robustness of this phenomenology to the introduction of dissipation". I would stress that also thermal effects are of interest here.

We now also refer to thermal effects here, and stressed the derivation of new Floquet CFT results at finite temperature and finite size.

- Section 2: "Using the tools developed in Ref. [47,48]". I think that the transformations in Eq. 2 were first given in this context in SciPost Phys. 2, 002 (2017) and SciPost Phys. 6, 051 (2019).

We thank the referee for pointing these references out. We added them to the manuscript.

- Section 4 (below eq. 27): $h_0 \to H_0$?

We thank the referee for this remark, but we decided to keep h_0, in order to stress that H_0 is an operator in the second-quantized language while h_0 is a finite-dimensional matrix.

- Section 4: About the extension of the Floquet-CFT results to finite temperature the authors claim "This complexity can be traced to the fact that for thermal states, the correlation functions cannot be fixed solely by conformal invariance, but require the knowledge of the full operator content of the theory at hand ". I honestly do not understand this point as, to my knowledge, zero and finite temperature correlations are simply related by a conformal transformation. I am probably missing something, but I think the authors could try to explain this better.

We stress that the Floquet CFT phenomenology only appears at finite system size L. Usual conformal transformations relate the Euclidean spacetime cylinder at finite size and zero temperature to the thermal cylinder at finite temperature and infinite size, while we want to have both finite system size L and finite temperature β^{-1}, giving a Euclidean space-time torus which cannot be mapped to a cylinder via a conformal transformation. This requires the knowledge of the full operator content of the theory at hand, as explained in the manuscript. See Sec. 3.1.1 in [1] for a thorough discussion of the question of finite size *and* finite temperature. However, we removed this paragraph in the current version of the manuscript as we actually managed to derive the finite temperature time-evolution of the energy from CFT at finite size, using the torus partition function of the compactified free boson CFT, together with the 2×2 representations of the $\mathfrak{su}(1, 1)$ algebra and the Floquet Hamiltonian, see Sec. 4.1.

References

[1] P. Calabrese and J. Cardy, "Entanglement entropy and conformal field theory," *Journal of Physics A: Mathematical and Theoretical*, vol. 42, no. 50, p. 504005, dec 2009. [Online]. Available: https://doi.org/10.1088%2F1751-8113%2F42%2F50%2F504005