Reply to report on [2112.10514]

Dear Referee:

Thank you very much for your patience and valuable comments and suggestions. We have modified our paper according to your report. In this reply we explain our modifications and answer your questions.

1. The states in the highest weight representation are the eigenstates of dilation operator D, and are annihilated by K. This structure is not a representation in Wigner classification should be. This can be seen in the discussion in $2d$ [1]. The representations from the Wigner classification (called Poincare module in [1]) are unitary and not of highest weight. We agree that the representations from Wigner classification could be useful for other purposes. We added a footnote in the Introduction to emphasize our focus.

2. $so(4,1)$ is a typo, and we changed it to $so(4,2)$. We corrected the statement below Fig 1, in accord with the discussion on the taking-limit method.

3. The n-point invariants only appear in "stripped correlators" as in CFT. The stripped correlators of 2-point and 3-point correlators are trivially 1, and the correlators we calculated are the dynamical parts. In other words, the n-point invariants only play a role in the correlators of $n \geq 4$ operators.

4. Although it seems straightforward to use Young tableau of $SO(d)$, it is more concise to show our algorithm by using the Young tableau of $SU(d)$. The reason is that the rank of the tensor product is equal to the number of the boxes by using the Young tableau of $SU(d)$, and such relation gets lost in using the Young tableau of $SO(d)$. For example, the rank-3 tensor representation is shown in Figure 1, and the relations among the representations are more transparent by using $SU(d)$ tableau. We added a footnote to clarify this point.

5. Thank you very much for this suggestion, and we added a statement on this relation at the end of Section 3.5.

6. We have updated (4.5) to be the same with (B.1).

7. We modified the statements below (4.11) (Eq. (4.14) in the new version) accordingly, and added a new paragraph below Eq. (4.15).

8. In fact, the representations other than singlet representation are equally important. For example, the Carrollian $U(1)$ gauge fields A [2] are in a chain representation, and it would be shown in a subsequent work that the stress tensor F is in a net representation. There are other theories in which the operators other than singlet representation have been discussed, see e.g. [3,4]. Furthermore, considering the operator product expansion of 4-pt function, the operators in all possible representations may appear in the propagating channel even if the external operators are in simple representations. We added a few sentences in the beginning of Sect. 3.3 to stress the importance of other complicated representations.

9. The type of correlation functions we focused on could appear in the bilocal scalar actions, which we showed in Eq. (4.11)(4.12) in section 4.1. Besides, in the $2d$ BMS free scalar model [5], the correlation functions of the highest weight vacuum are also fitted into this type. For Galilean CFT, the bilocal actions can be constructed similarly, and in $2d$ the BMS free scalar model also provides an example due to the coincidence of $2d$ Galilean and Carrollian conformal symmetries.

References

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[3] A. Bagchi, A. Mehra and P. Nandi, *Field Theories with Conformal Carrollian Symmetry*, *JHEP* **05** (2019) 108 [1901.10147].

[4] K. Banerjee, R. Basu, A. Mehra, A. Mohan and A. Sharma, *Interacting Conformal Carrollian Theories: Cues from Electrodynamics*, *Phys. Rev. D* **103** (2021) 105001 [2008.02829].

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