

## Reply to report 2 on [2112.10514]

Dear Referee:

Thank you very much for your patience and valuable comments and suggestions. We have modified our paper according to your report. Here we explain our modifications.

1. Thank you very much for this suggestion. We added a sentence on the BMS-like extension of Carrollian symmetry at the end of Page 3, and included the references.
2. The Casimir operators of GCA can be obtained by taking the non-relativistic limit. We have corrected the statement in page 9.
3. We added a brief comment on the magnetic sector of free Carrollian scalar in the paragraph below equation (4.15). We will give a more detailed discussion on the electric/magnetic sector of free Carrollian scalar in an upcoming paper.
4. Generally, there are electric and magnetic sectors of Galilean field theories, similar to the Carrollian case. But for Galilean free scalar, there is only the electric sector with  $\mathcal{L}^M = \delta^{ij} \partial_i \phi \partial_j \phi$ , and there does not exist the magnetic sector. Following [1], we start from the action

$$S = \int dt \left[ \int d^d x \pi \partial_t \phi - \int d^d x \frac{1}{2} \left( c^2 \pi^2 + \partial_k \phi \partial^k \phi \right) \right].$$

By taking  $c \rightarrow \infty$  limit, we find the following actions,

magnetic sector:

$$S^M = \lim_{c \rightarrow \infty} \int dt \left[ \int d^d x \pi \partial_t \phi - \int d^d x \frac{1}{2} \left( c^2 \pi^2 + \partial_k \phi \partial^k \phi \right) \right] = c^2 \int dt d^d x \pi^2$$

electric sector:  $\phi = \frac{1}{c} \phi'$   $\pi = c \pi'$

$$S^E = \lim_{c \rightarrow \infty} \int dt \left[ \int d^d x \pi \partial_t \phi - \int d^d x \frac{1}{2} \left( \pi^2 + c^2 \partial_k \phi \partial^k \phi \right) \right] = \frac{c^2}{2} \int dt d^d x \partial_k \phi \partial^k \phi$$

The magnetic one has a trivial action since we can integrate out the momentum  $\pi$ , while the electric sector is the well known Galilean scalar theory.

The correlator of the electric sector is easy to calculate and proportional to  $\delta(t)$  as expected. We will put this discussion in the upcoming paper rather than in this paper.

5. We corrected the typos pointed out in the report.

## References

- [1] M. Henneaux and P. Salgado-Rebolledo, *Carroll contractions of Lorentz-invariant theories*, *JHEP* **11** (2021) 180 [[2109.06708](#)].