Response to the comments of the First referee

We thank the referee for finding our study interesting and worthy of publication in SciPost Physics. Below we present our response to the specific comments of the first referee.

1) The first matrix product solution for a two species ASEP was given in Derrida, B., Janowsky, S.A., Lebowitz, J.L. , Speer E.R.. Exact solution of the totally asymmetric simple exclusion process: Shock profiles. J Stat Phys 73, 813–842 (1993). https://doi.org/10.1007/BF01052811. (I will refer to this paper as [DJLS] below.) The work [DJLS] should certainly be cited.

We thank the referee for pointing out the above mentioned reference. This has been cited as new reference [25] (in the introduction and the discussion of matrix representations) in the revised manuscript.

It also showed how the stationary state factorises about the defect (a 'second-class' particle in that case) which implies a projector form for the matrix A, the same as in equations (10) and (13) of the present work. This factorisation property due to the projector form of A should be acknowledged in the current paper.

We thank the referee for this indication. We refer to the above mentioned article in context of the projector form of the impurity, after Eq. (10) of the revised manuscript. The fact that the stationary state factorizes about the impurities, has now been stated with the new Eq. (23) in the revised manuscript.

2) I am not sure I understand the last sentence of Section 2 'However, we should mention that.. this is not the general expression for α ..'. Does this mean that generally α would appear as α_I in equation (15)? Perhaps this point can be clarified.

Ye, it does. As the referee correctly pointed out, we indeed meant that the parameter α appearing in Eq. (15) for $\mu = 3$ (and $\alpha = 0$ for $\mu = 2$), generalizes to α_{KI} for $\mu > 3$, i.e. α_{KI} can explicitly depend on the pair (K, I) of the species. We have stated this fact with the new Eq. (18) in the revised manuscript and to clarify the situation with an example, we have added a new figure (Fig. 2) along with a brief discussion after Eq. (18). This basically illustrates a specific example of

$\mu = 4$ where certain flip rates are absent, leading to the generalization $\alpha \equiv \alpha_{KI}$.

3) Second sentence of Section 3. I would put transfer matrix in inverted commas as this is not the same as a usual equilibrium transfer matrix. i.e. 'Here the "transfer matrix" T refers to ..'

We thank the referee for this indication. Accordingly, the above mentioned sentence has been changed to 'Here the "transfer matrix" T refers to' before Eq. (19) in the revised manuscript.

4) In section 3 the grand canonical ensemble is used , which results in a fugacity z_0 which is fixed by the density. It would be helpful to have the solution for z_0 appear somewhere, perhaps in an appendix if it is really very complicated. Does the solution for z_0 simplify in some limits?

Yes, it does. In general it is difficult to achieve closed form expression for the fugacity z_0 from the density-fugacity relation. However, for special subspaces of the whole parameter space, it might be possible to get closed form solutions. In this connection, we have added a new Appendix D in the revised manuscript to discuss the solution for z_0 for two special cases in the reduced parameter space where the actual density-fugacity relation reduces to quadratic and cubic equations respectively, leading to closed form solutions for z_0.

5) Section 5.4 considers Negative Differential Mobility (NDM). Some references and discussion of specific, related models, exhibiting NDM would be appropriate e.g. Cividini J, Mukamel D and Posch H A "Driven tracer with absolute negative mobility" (2018) J. Phys. A: Math. Theor. 51 085001 .

We thank the referee for this suggestion. In the end portion of the first paragraph in Section 5.4, we have added a short discussion on the variants of asymmetric simple exclusion process where NDM have been observed by implementing kinetic constraints either by making the motion dependent on the occupation of neighbors of the departure and arrival sites, or by choosing the escape rate from a configuration as a decreasing function of the bias. The above mentioned reference has been cited in this discussion as a new reference [84] in the revised manuscript.

6) Some typos: p.3 paragraph 2 'with variety' 'with a variety'
p.4. paragraph 2 'For specific choice of' 'For a specific choice of'

p.5 last line ‘itlaics’ ‘italics’

p.7 after equation (10) ‘resembles to that of the defect or second class’ ‘resembles that of the defect of second class’. Here reference [DJLS] (see point 1. above) should be cited.

p.36 reference [59] author is J. Szavits-Nossan.

We thank the referee for identifying the typos. The typos have now been corrected in the revised manuscript.