Reply to Referee 4

We thank Referee 4 for recommending publication of this article in SciPost. Thank you for useful suggestions and comments, which have helped us to improve the manuscript and its readability substantially.

1. **Q:** It took me some time to realize that the third update in (1) allows only the left-hand particle to change direction ("tumble"), and then only when the right neighbour is moving the same way. I think it would help first of all to write out the ω_+ and ω_- rules separately, and furthermore to state clearly in the text which updates are allowed and not allowed.
   Ans: The tumble dynamics is now given as a separate equation and the constraint is described in length.

2. **Q:** Related to 1, I think some discussion of the rationale for this set of rules is required. I understand that this allows analytical progress later in the manuscript, but did not follow at what point this becomes important. Furthermore, one may worry that the rule breaks CP symmetry for those cases where $r_+ = r_-$ for each of the transition rates $r \in \{p, q, \omega\}$ and such symmetry might be expected. I understand that setting certain tumbling rates to zero moves in the direction of making the system more likely to phase separate, so I doubt this affects the final conclusion, but one does feel nervous about removing a symmetry from a model that might initially be present.
   Ans: Thank you for mentioning - it is indeed a very important point. Although the model is defined in a general set up, while considering RTP particles we have set $p_+ = q_-$ and $p_- = q_+$ and $\omega_+ = \omega_- = \omega$ which indeed keeps CP symmetry preserved. This condition $p_\pm = q_\mp$ is same as the same speed criteria $p_- - q_- = v = q_+ - p_+$ we mentioned earlier (it is also mentioned in Ref. [22]). In the revised version, CP symmetry is discussed explicitly.

3. **Q:** I did not follow the set-up of the coarse-grained model at all. The exact urn model has particles hopping in both directions, whereas the coarse-grained model seems to have them moving only to the right (or perhaps this is a misleading impression given by Fig 1c)? I did not see how the hop rates $u(m)$ are defined, whether these depend only on the occupancy of one site (as suggested by Fig 1c) or two sites (as suggested by the text). I really could not understand why some of the domains of size m in Fig 1b overlap, or what determines the start and end of a domain. This entire mapping needs to be laid out completely and methodically.
   Ans: We have modified Figure 1. In Fig. 1(a) we describe the exact mapping of the RTM and Urn model - here urns have internal degrees (spins) and rates do not depend on the number of beads but depend on the spins of arrival and departure urn. We then provide a coarse-grained dynamics of this urn model, by integrating the spin of arrival and departure urns - but the rates are assumed, in general, to depend on the beads present in arrival and departure urn. Mapping the model to misanthrope process, instead of zero range process is relevant as the dynamics of the original urn model was dependent on the properties (spin) of both arrival and departure urns.

4. **Q:** In Fig 2, where hop rates are obtained from numerical simulations, it was unclear what was being simulated here. Is it the full dynamics of the original model, or one of the derivatives described in the main text.
   Ans: Numerical simulations of full dynamics given by Eqs. (1) and (2) are studied here. We clarified it in the revised version.

5. **Q:** I would like to see a bit more discussion of the comparison with Ref 27. Although the latter relies on a specific limit, I understood that the current analysis applied to any parameter combination so one ought be able to take the same scaling in the present work to compare more directly. Moreover, the $1/L$ separation of the run and tumbling rates I think corresponds to the scaling limit considered in Ref 15, which makes me wonder if something different happens in this scaling limit and whether this can be seen from the present analysis.
   Ans: We think in Ref [17] (in the revised version) $1/L$ scaling limit came from the fact that there are only two RTPs and the typical distance between them is $\sim 1/L$ and the density of the system is also $\sim 1/L$. On the other hand if tumbling rates are $\sim 1/L$ then, tumbling can not win over the run dynamics in the thermodynamic limit and one may encounter a phase separated state (as the system absorbing/fully-jammed configurations when $\omega = 0$.)

Summary of changes

The article has gone through a **major revision** following the valuable comments of the referees. It is difficult to provide details as in the revised manuscript, the whole structure and almost all the paragraphs are modified. In the following we list some important changes.

1. Restricted tumbling dynamics is now given separately as Eq. (2) followed by a longer discussion.

2. Fig. 1 is modified - exact and coarse-grained urn models are now described more clearly in Fig1(b).

3. Discussions on Matrix Product Ansatz (MPA) is described in the APPENDIX. We hope it helps the readers to arrive at the results and conclusions of the article without bothering much about the detailed mathematical steps of MPA.

4. New references [15], [16] and [23] are added.