# Reply to report on [2112.10514]

Dear Referee:

Thank you very much for your comments and suggestions. We have modified our paper according to your report. In this reply we explain our modifications and answer your questions.

4. To make the exposition more clear, we made major revisions in section 3.2, starting from the last paragraph of page 18 to the end of section 3.2. The main idea is to introduce an algorithmic method to decompose the tensor representations $V^{\otimes k}$ into indecomposable (but reducible) representations of Carrollian rotations $ISO(d-1)$: $V^{\otimes k} = \bigoplus_{n=1}^{N} V_n$, where $V$ is the vector representation and $V_n$ are indecomposable. There are roughly two main steps.

   Firstly due to the inclusion $ISO(d-1) \subset SL(d)$, we use $SL(d)$, or equivalently $SU(d)$ Young diagrams to decompose the tensors $V^{\otimes k}$ into irreducible representations of $SL(d)$. Then from all the examples we find that these $SL(d)$ irreducible representations are kept indecomposable under the inclusion $ISO(d-1) \subset SL(d)$. We believe this is true and stress in the paper this is merely a conjecture.

   Secondly motivated by the consideration of vector representation in section 3.2.1, we want to study the structure of the tensor representations $V_n$ as $ISO(d-1)$ indecomposable representation, by breaking it into $SO(d-1) \subset ISO(d-1)$ irreducible representations $V_n = \bigoplus V_{n,i}$, and recombining $V_{n,i}$ via the boost generators. By the first step we need to consider the $SL(d)$ Young diagram, and find that each $V_{n,i}$ corresponds to a $SL(d)$ Young diagram together with spatial, temporal and trace-free projectors. This is done by filling the boxes in $SL(d)$ Young diagram with spatial or temporal indices, and bookmarking the index contractions.

   Since each $V_{n,i}$ is an irrep of $SO(d-1)$, after computing the decompositions we can also replace those generalized Young diagram in step 2 equivalently by $SO(d-1)$ Young diagrams. This is not necessary and will lose track of the projection and contraction of indices. We briefly compare the advantages and disadvantages of our notations with $SO(d-1)$ Young diagram.

We wish we have clarified why use the Young tableau of $SU(4)$ (or $SL(4)$). The discussion in this subsection is not essential - it is just to give a flavour on the representation of CCA rotation, inspired by the study from taking the $c \to 0$ limit. The more general discussion on the representation starts from the section 3.3.

7. The discussion in [2207.03468] is interesting, but could miss something important. We think the statement in our paper is correct.

   In that paper, the authors integrated out the field $\chi$ and argued that the theory simply reduces to a lower-dimensional one. This treatment leaves out some solutions and needs more careful investigations. From our point of view, the field $\chi$ is actually dynamical, and should not be integrated out. Its dynamics can be seen, once we add a total derivative term. Concretely, the Lagrangian of magnetic scalar in flat space is

$$
\begin{aligned}
\mathcal{L} &= \int d^d x \; 2\chi\partial_0\phi + \partial_i\phi\partial_i\phi \\
&= \int d^d x \; 2\chi\partial_0\phi + \partial_i\phi\partial_i\phi - \partial_0(\chi\phi) \\
&= \int d^d x \; \chi\partial_0\phi - \partial_0\chi\phi + \partial_i\phi\partial_i\phi.
\end{aligned}
$$

   Using the path integral, we find the correlators indeed have $\delta$-function structure satisfying our restrictions:

$$
\langle\phi\phi\rangle = 0, \qquad \langle\phi\pi\rangle \propto \delta(\vec{x}), \qquad \langle\pi\pi\rangle \propto t\delta(\vec{x}),
$$

   where $\vec{x}$ are $(d-1)$-dimensional spacial vectors. On the contrary, in [2207.03468], the Lagrangian after integration and the resulting correlator in flat space are respectively

$$
\mathcal{L} = \int d^{d-1}x \; \partial_i\tilde{\phi}\partial_i\tilde{\phi}, \qquad \left\langle\tilde{\phi}\tilde{\phi}\right\rangle \propto \frac{1}{\vec{x}^2},
$$

   where $\tilde{\phi}$ is the redefined scalar.

   One way to understand this difference is the following. Usually one chooses the field configurations in the path integral to be the ones fast decaying at infinity, e.g. Gaussian wave packet

$$
\phi(t,\vec{x}) \xrightarrow{t\to\pm\infty} 0.
$$

   While the process of integrating out the field $\chi$ imposes the constraint $\partial_0\phi = 0$ on the field $\phi$. This means that in [2207.03468], the field $\phi$ is of the form of a pillar wave packet in $t$ direction:

$$
\phi(t,\vec{x}) \xrightarrow{t\to\pm\infty} \phi(\vec{x}),
$$

which is generally non-vanishing. Thus these two methods take account of different field configurations, resulting in different correlators. From the perspective of quantization, different field configurations correspond to different choices on the vacuum and quantization scheme.

We will discuss the magnetic scalar theory $\mathcal{L}$ and its difference from the one in [2207.03468] in detail in an upcoming paper. Thus we do not put these discussions in the current paper.