Response to referee 2

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1.

> In the summary above I spoke as if there were a single boson. Actually the bosons in the paper come with two indices ϕ_I^a. The a index is eventually dropped as not necessary. Is the I-index on the boson necessary?

This is a very interesting question to which we do not have a complete answer. On the one hand, in order to get a controlled set of large N saddle point equations for ϕ_I and ψ_I, we need N flavors and bosons and fermions. In that sense, the I-index is necessary. On the other hand, even without the I-index, random flavor interactions of the form $g_{IJ}\phi\psi_I^\dagger\psi_J$ are sufficient to invalidate the anomaly constraints for the $N = 1$ model and possibly allow for a nontrivial incoherent conductivity. Such a deformation also avoids the issue of multi-criticality in the boson action. It would be very interesting to understand if anything analytic can be said about the frequency dependence of $\Pi(\mathbf{q} = 0, \omega)$ and $\sigma(\mathbf{q} = 0, \omega)$ in this deformation.

2.

> I didn't like the paragraph after equation (3.6), about the regulator-dependence of the expressions for various densities in terms of the fields. The reader has just been introduced to these quantities n_θ and $\tilde{n}_\theta$ and to immediately be told that the concrete expressions involving them are so mercurial is disheartening. It would be nice to rephrase this paragraph to first say more things that are robustly true about these quantities.

This is a great point. We have rephrased this paragraph to highlight the physical meaning of these anomaly equations as deformed Ward identities and relegated the comment about regularization dependence to a footnote.

3.

> At various points there is a close parallel between the coupling to a loop current order parameter and to an abelian gauge field. I understand that this is a target for future work, but it might be nice to comment about where the analyses depart. (For example, does the discussion on page 16 apply to the gauge field case?)

Indeed, most parts of our analysis apply equally well to loop current order parameters/$U(1)$ gauge fields (including the discussion on page 16). We have added some comments to emphasize this fact.

4.

> The paper, as a sort of bonus, contains a reanalysis of the calculation of Ref 64, which regards the inclusion of $O(N_f)$-symmetric couplings of N_f Fermi surfaces to a gauge field, including irrelevant couplings. As currently presented, this might be a bit of a distraction from the main focus of the paper. I think it is a nice thing to include, but perhaps more can be done to sequester it.

Thanks for this suggestion. We have revised Section 3.4 to sequester our reanalysis of Ref. 64 and highlight our findings.