Response to the report of the Reviewer-2

We greatly appreciate the constructive remarks and the suggestions made by the referee. These remarks are addressed as follows.

1. could be written slightly more clearly, also it is unclear what this approach gives more than NLFH

> **Response** : We would like to emphasize that the late time super-diffusive evolution can indeed be obtained by directly following the procedures of the NFFHD. In our approach, we have indeed used fluctuating hydrodynamics to compute the current-current space-time correlation that finally contributes to the non-local kernel. The main difference is that our starting point is different from the NLFHD theory. We have started from the FP equation and seek solution of it that is in the LE form and always remain close to an underlying GE state. As usually done in fluid hydrodynamics, we compute the averages of the conserved fields and the associated currents (related via continuity equations) with respect to these solution. Invoking certain physical assumptions, we have demonstrated that the contribution from the deviations from the LE state to the average current indeed comes from the time-integral of un-equal time correlations of (local) currents at different locations. Note such local current-current correlations are not usually studied in the NLFHD theory, instead one often studies the total current-current correlation. The particularly simple (linear) form of the fluctuating equation satisfied by the volume field in our model allows us to compute this correlation and its time integral analytically, both for the closed (without reservoirs) and open-system set-up. We have added the above discussion on page 11 (the paragraph after Eq. (37)] .

2. I would require the author to clarify in the abstract that what he found is an agreement with NLFH and also to clarify why he thinks that NLFH cannot provide the crossover from diffusion to superdiffusion.

> **Response** : We thank the referee for the remark. As was already mentioned in the previous version (page 3 2nd last paragraph), one expects to find a crossover from diffusive to super-diffusive evolution within NLFHD itself. In fact such a crossover has been studied by considering diffusing correction to the leading mode-coupling solution of the correlations in the NLFHD in Ref [19].

In the modified version we made the discussion clearer by commenting on the recent work (Ref. [19]) in which the diffusive correction to mode coupling solutions were studied. We also have added a sentence in the abstract in this regard (see at the end of page 3).

3. - Could the author write a clear and full derivation of eq. 15? It should be clear that the solution for P_d is in the linear response.

Response : The solution of Eq. (15) [now Eq. (14) in the revised version] can be written as follows: The equation can be recast as

$$\partial_t P_d(t) - \mathcal{L} P_d(t) = \Psi(t), \tag{1}$$

where $\Psi(t) = [\Phi(t) - \Phi_{LE}(t)] P_{LE}(t)$ is a source term in the equation and $\mathcal{L}$ the FP operator. Then for the initial condition $P_d(0) = 0$, it is easy to write the solution formally as

$$P_d(t) = \int_0^t dt' e^{\mathcal{L}(t-t')} \ \Psi(t'). \tag{2}$$

Later when we used this solution in sec. 3.2, we have assumed that the deviation P_d to the LE distribution is in fact small as we are interested to find the linearised hydrodynamics. We have now added a discussion clarifying what we mean by P_d small and clearly stated the assumptions that are required for the derivation to go through. Formally the solution in Eq. (11) is exact. However, since we are interested in linearised hydrodynamics, it is sensible to assume that the deviations from the global equilibrium characterized by $\tilde{T}_i(t) = T_i(t) - T_0$ and $\tilde{\tau}_i(t) = \tau_i(t) - \tau_0$ and their space-time variations are small so that the system always remains close to a LE state which is slightly deviated from the GE state. Equivalently, one can say that the deviation P_d, which depends on $\tilde{T}_i(t)$ and $\tilde{\tau}_i(t)$ and their space-time derivatives (see Eq. (16)), is also small. These assumptions, are used later in sec. 3.2 [see discussions between Eqs. (30) and (32)], where we neglect terms involving higher order in deviations as well as higher order in derivatives.