Answer to first referee report

November 5, 2018

1st comment: In the discussion of their results, the authors should provide a more concise description of the physical situation under consideration. In particular the precise initial conditions of their non-equilibrium dynamics remained unclear to me. On the bottom of page 18 the authors state that they start from a state without entanglement. However, in Fig. 1, they present results showing how there is entanglement present at intermediate times, which disappears. How does the entanglement evolve at shorter times? Is there some entanglement present when the dynamics are started?

Our reply: First there is a small typo. Equation (86)

$$x_j(t) = \sum_{q=1}^{2} x_q(0) + \alpha_{sq}\dot{x}_q(0)\,t + \int_0^t (t-s)\,\alpha_{jq}B_q(s)\,ds$$

should have actually been

$$x_j(t) = x_j(0) + \sum_{q=1}^{2} \alpha_{jq}\dot{x}_q(0)\,t + \int_0^t (t-s)\,\alpha_{jq}B_q(s)\,ds.$$

With this in mind, the initial conditions are the following: we assume that the state of the impurity and the BEC are initially uncorrelated such that:

$$\rho(0) = \rho_I(0) \otimes \rho_B,$$

where $\rho(0)$ initial total state of system-bath configuration, $\rho_I(0)$ the initial state of the impurities and ρ_B the thermal state of the bath. Regarding the initial state of the impurities, the initial state assumptions made, without loss of generality, were:

$$\langle x_j(0)\,\dot{x}_q(0)\rangle = 0 \qquad j \neq q,$$

$$\langle \dot{x}_j(0)\,\dot{x}_q(0)\rangle = 0 \qquad j \neq q,$$

and

$$\left\langle (\dot{x}_j(0))^2 \right\rangle = 0.$$

We examined also the case were both of these quantities, i.e. $\langle x_j(0)\,\dot{x}_q(0)\rangle, \langle \dot{x}_j(0)\,\dot{x}_q(0)\rangle$ and $\left\langle (\dot{x}_j(0))^2 \right\rangle$ had a finite value (we set them equal to 1), but no qualitative difference was observed in the results.

We were not able to study entanglement evolution for earlier times than the ones presented, since at those times, uncertainty principle which is examined by the condition given in Eq. (77), was not satisfied. Furthermore, the results of our approach refer to the long time limit considered in obtaining Eq. (82) and in accordance with the studies undertaken in [2], in order to be able to obtain our analytical results for the untrapped particle case. To be able to study the short time limit, one would need to resort to numerical studies using the Zakian method as explained in [2].

According to this discussion, an amendment has been introduced in the text at the end of page 18, just before Eq. (93).

2nd comment: The authors should provide some discussion how the quantities they calculate can be measured experimentally. For example, the entanglement is characterized by Eq. (73), where \nu_- is determined as "the smallest symplectic eigenvalue of the partial transpose covariance matrix C". How can one, at least in principle, measure such a quantity? Similar, abstract properties of the covariance matrix are required in Eq. (79) to characterize squeezing in the system.

Our reply: There are two kinds of terms that appear in the covariance matrix that one should evaluate, single particle expectation values, such as $\langle x_j^2\rangle, \langle p_j^2\rangle$, and crossterms such as $\langle x_1 x_2\rangle, \langle p_1 p_2\rangle$. For the former, there are already experiments in which one is able to evaluate them [3]. The idea is that one measures the position (or momentum) of the particle using a time of flight experiment in a system with a two species ultracold gas, in which one of the species is much more dilute - dilute enough as to consider its atoms as impurities immersed in a much bigger BEC. The position variance is obtained from the time-of-flight experiments, by releasing the atoms into free space initially and after allowing for the free expansion of their wavefunction for some time, measuring their position by irradiating them with a laser. Nevertheless, this method is not ideal for obtaining the real space information of a trapped sample, since during the free expansion process, signals from other atoms can easily be mixed with the signal of the atoms of interest. Furthermore, the current status of time-of-flight experiments does not allow for the measurement of the crossterm covariances, which as of now there are no experiments to measure them, but we believe that one should be able in principle to do so.

In particular, a quantum gas microscope [4, 5] might be an option. This technique uses optical imaging systems to collect the fluorescence light of atoms, and has been used in the study of atoms in optical lattices, achieving much better spatial resolution [4, 5], and avoiding the aforementioned problem with time-of-flight experiments. In the past such a technique has been used to study spatial entanglement between itinerant particles, by means of quantum interference of many body twins, which enables the direct measurement of quantum purity [6].

In addition, there is an alternative way to study entanglement in continuous vaiable system, and that is by means of the average logarithmic negativity [7]. This quantity, can be related to the global and local purities of our system, which are measurable quantities. Nevertheless this is a more brute measure, since it is not estimated directly from the state of our system, and hence it may miss to detect entanglement for the actual state of our system.

Even if this is the case for average logarithmic negativity, with the knowledge of the existence of this measure and the technique of quantum gas microscopes, one could still consider this way of measuring entanglement as the worst case scenario. In general, it is known that to measure entanglement in a system of continuous variables is a difficult task, which is a problem that is not restricted to our case.

An amendment was introduced in the text in the second paragraph of pg. 15.

3rd comment: In the beginning of Sec. 4, the authors introduce a condition where quantum effects play a role, which relies on a comparison of the temperature T to the trapping frequencies. I find this confusing, since I would expect quantum effects to play a role even in a homogeneous system when \Omega = 0 vanishes.

Our reply: The condition for the case of harmonically trapped particles is indeed

$$k_B T \ll \hbar\Omega_j.$$

However, this condition is not applicable for the case of untrapped particles $\Omega_j = 0$, and we would like to thank the referee for pointing this out. In this case one would distinguish the high and low temperature limits using the condition

$$k_B T \ll \hbar\Lambda$$

where $\Lambda = g_B n_0/\hbar$ is the frequency cutoff identified in Eq. (48). Here, $\hbar\Lambda$ is hence understood as a chemical potential for the bath. The condition is then requiring that the equipartition energy of the free particle should be much smaller than the chemical potential of the bath.

An amendment was introduced in the text at the beginning of subsection “Out-of-equilibrium dynamics and entanglement of the untrapped impurities", pg. 17.

References

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