Re: Report 1 submitted on 2022-12-05 on scipost 202211 00007v1.

Title: "Quasi-localized vibrational modes, Boson peak and sound attenuation in model mass-spring networks"
Authors: Shivam Mahajan and Massimo Pica Ciamarra

Dear reviewer,

We made appropriate changes according to your suggestions.
We include in the following a detailed answer to all the remarks, and the list of changes.

We hope that our revised manuscript is now fully suitable for publication in Sci Post, and thank you for your consideration.

Your sincerely,
Massimo Pica Ciamarra

List of changes:

1. We revised a sentence in the introduction for clarity on the predictive abilities of FET and added the citation we inadvertently missed.
2. We have added a novel study (suggested by referee 2) to better characterize the spatial correlation of the local elastic properties. This study clarified that their correlation length is not affected by our algorithm. We have changed the manuscript to explain and highlight this point.
3. We have investigated the effect of the prestress. We have found network with the same prestress may have different elastic properties, e.g., disordered parameters and QLMs properties.
4. We have revised the discussion on sound's attenuation as suggested by the reviewer.

Referee #1:
I would be happy to recommend this paper as a publication in scipost if the requested changes are addressed.

> **Reply:** We thank the reviewer is willing to recommend our manuscript. We address the reviewer's remarks in the following.

Major comments:
The statement that the proposed algorithm does not affect pre-stress is not entirely true. Yes I agree that in percent it is relatively small (about 3-5%) but parallel works have shown that this amount is already enough to significantly decrease the fluctuations of the shear modulus. For example in [Ref.45 & arXiv:2012.03634], authors have reported a change of γ about a factor of 9 (factor 3 in what they defined as χ with $\gamma = \chi^2$) for a change of prestress around 5%. This result is fully consistent with the numbers reported in the present paper. I would therefore change the statement that the algorithm does not affect prestress, as they have a deep connection with the decrease of ξ_e and γ.

> **Reply:** The reviewer is certainly right when clarifying that our algorithm slightly affects the prestress. We have amended the paper to clarify this point.
>
> As the reviewer emphasizes, both in previous works and in our manuscript there is a striking contrast between minute prestress changes and large variations in the elastic properties. This observation, and the possibility of changing the the vibrational properties of mass-spring networks without affecting the prestress, e.g., by adding to the network unstressed springs, suggested to us that vibrational properties and prestress are not always deeply related.
>
> We have performed additional investigations to clarify if the changes in the vibrational properties of our systems originate from those of the prestress. After performing our bond-swapping algorithm, that leads to a reduction Δe in the pres-stress and to changes in the vibrational properties, we compress the system to eliminate the change in pre-stress. We find the vibrational properties of systems prepared via bond-swapping + compression ($\Delta e = 0$) to essentially coincide with those of system only prepared via bond-swapping ($\Delta e \simeq 3-5\%$). This finding clarifies that, at least in our model system, changes in the vibrational properties do not originate from changes in the pre-stress as we originally speculated. It would be certainly of great interest to perform this study also in the models described in the papers the reviewer mentions.
>
> We have included this additional study in the main manuscript.

I would revise the discussion about the correlation length associated with the shear modulus spatial distribution and its connection with γ, BP peak, and wave attenuation rate. It was previously established in the literature that the length scale associated with the core of QLMs $\xi_{qlm} \sim c_s/\omega_{qlm}$, with ω_{qlm} a characteristic frequency for localized soft modes [Ref.16]. It was speculated that the same frequency is matching the location of the BP peak, which was recently demonstrated using a non-linear framework [arXiv:2210.10326]. This glassy length was also shown to scale with the disorder parameter γ as $\xi \sim \gamma^{1/d}$ [González-López, Karina, et al. PRE 103(2021)]. The correlation length of the shear modulus distribution is simply the core size of QLMs (there are no other mesoscale length scales in amorphous solids). It explains why $\xi_e \sim \gamma^{1/2}$ and $\omega_{BP} \sim 1/\xi_e$ (as shown in the present paper and in Ref. 41 in 3d). This paper would gain in clarity with a dedicated discussion making the direct link between ξ_e and ξ_{qlm}.

> **Reply:** The reviewer's comment, and a related comment by reviewer 2, suggested a more detailed data analysis of our data which we report in the revised manuscript. This analysis indicates that our bond-swapping algorithm does not influence any microscopic correlation length ξ. Consistently with the reviewer's suggestions, we find both the correlation length of the shear modulus and the core size of QLMs to be constant. In the presence of a constant

elastic correlation length, the Boson peak frequency increases with the disorder parameter γ, in agreement with the prediction of correlated fluctuating elasticity. We have revised the discussion on the relationship between sound attenuation and QLMs for clarity. These results suggest sound attenuation may have dimensionality-dependent features reflecting those of the QLMs, we are currently investigating.

The original network used in this study is constructed from glasses featuring only repulsive interactions at a finite positive pressure. Could authors check if the decrease in ξ_e with the fraction of bond swapping f is also observed in glasses with attractive interactions prepared at zero pressure.

Reply: As mentioned in the previous comment, our more detailed analysis of the data reveals that bond-swapping does not affect the length scale ξ_e. Contrasting literature results may be rationalised if the elastic length scale sensibly varies in three dimensions, not in two. Accordingly, rather than exploring different interaction potential, we have started exploring bond-swapping algorithm in three spatial dimensions and hope to report interesting results soon.

Authors should put in perspective their results on the prediction of FET for sound attenuation with recent studies (e.g. Szamel & Flenner, JCP, 156 (2022)) showing that FET largely underestimates wave attenuation rates.

Reply: We thank the reviewer for his comment and for sharing the relevant literature, which he had inadvertently missed in our manuscript. We have revised our article and commented on the predictive abilities of FET.

Minor comments:

Citation 17 in the introduction is probably a mistake and should be replaced by citation 41 (same authors) where A_4 is reported for swapped Monte Carlo glasses.

Reply: We thank the reviewer for pointing out this error and have changed the citation.

In Equation 2, I would define explicitly for non-experts what is σ_W.

Reply: We thank the reviewer for his comment and have revised the manuscript to clarify this point.

page 5 after "to evaluate per-particle elastic constants" there is a full stop instead of a comma.

Reply: We thank the reviewer for highlighting the typo and fixed it.

Change "localised" to "localized" page 2

Reply: We thank the reviewer for highlighting the typo and fixed it.