Response to Referee #3

1. *This paper offers a way for designing Floquet driving protocols as an optimization problem, maximizing an observable in the nonequilibrium steady states (NESSs). Taking advantage of the simple structure of the Lindblad equation, the authors obtain gradients for the density matrix in terms of driving parameters, which allows the optimization of the NESSs.*

   Roughly speaking, this work is a direct combination of Ref. 28 and Ref. 36. Reference 28 discusses the optimization problem without Floquet drivings, whereas Ref. 36 discusses the NESS without optimization. In this view, the current title, "Floquet engineering non-equilibrium steady states," is inappropriate and too general since this concept was already proposed, at least in Ref. 36.

   Besides, the paragraph "However, ..." in the Introduction is referenced only by Ref. 28, and the authors completely ignore all the relevant works on Floquet engineering in open systems, including the NESS, even stating, "we extend here that previous concept of Floquet engineering to open-quantum systems." The authors' contribution is not to extend Floquet engineering to open systems but to develop an optimization technique for (previously proposed) Floquet engineering of NESSs. I strongly recommend that the authors read a recent review article, arXiv:2203.16358, and revise the Introduction and title so as to highlight their contribution correctly.

   We would like to thank the referee for valuable suggestions to improve the manuscript. We agree with the referee that extending the concept of the Floquet states to nonequilibrium steady states under dissipation has already been proposed even before Ref. 36, for example, Ref. 19. However, we do not agree with the characterization of this work as a direct combination of Ref. 28 and Ref. 36.

   (Note: the references numbers have changed in the new revision, the numbers 28 and 36 and 19 refer to the previous one)

   First of all, the method used in Ref. 28 is not applicable to the situation that we tackle in our work. The method that we propose is applicable to time-dependent periodic drivings. We have discussed this issue in the answer to the other reports, and therefore we do not repeat it here (see the response to report number one). We have also added a discussion about this issue in the text.

   Second, we would like to comment on the notion of "engineering": "designing and/or building something useful/desired based on scientific principles". We are probably using that notion with a meaning that goes beyond what the referee implies. Generally speaking, we think that a simple parameter scan is not engineering; more complex controlling procedures are required, such as the optimal control theory. We therefore use the word engineering with a meaning that is closer to "optimization of properties of processes". In this respect, to our knowledge, the present manuscript is the first trial to engineer the desired material properties/functionalities on demand for non-equilibrium steady states under periodic drivings – beyond merely conducting parameter scans with respect to the drivings. In this work, we provide an explicit answer to how one can "engineer" the nonequilibrium steady states to realize desired properties/functionalities by controlling light fields. Previous works have studied the emergence of Floquet driven NESSs, have studied procedures to compute them, and have scanned them with respect to field parameters – without taking the further step of optimization.

   In any case, we agree with the referee that the title is too general: to emphasize this point, we propose the following revised title, "Floquet engineering non-equilibrium steady states: on the optimization of system properties with gradient-based methods"

   We also agree that the sentence "... we extend here the previous concept of Floquet engineering..." was ambiguous in respect to what that "previous concept" referred to. In order to clarify this point in the corresponding paragraph in the introduction, we have revised that text as "... we extend here that previous concept of Floquet engineering based on QOCT to open-quantum systems."

   We also thank the referee for pointing towards arXiv:2203.16358 (already published in Annual Reviews of Condensed Matter). We have added that reference to the text.

2. *Given that this work is a direct combination of Refs. 28 and 36, I do not find it a groundbreaking discovery or a breakthrough that is required in the acceptance criteria. So I cannot recommend publication in this journal. Since the technical results seem valid and interesting, I recommend publication in a non-flagship journal like SciPost Physics core after making appropriate revisions.*

   We thank the referee for appreciating our work as "valid and interesting", and worthy of SciPost Physics Core. However, we would like to request a publication in the flagship journal SciPost Physics. We hope that the novelty and significance of the present work have become more clear in the revised manuscript. We believe that

our work meets the criterion for publicaction in SciPost Physics. In particular, we think that it fully fulfills one of the expectations: “Open a new pathway in an existing or a new research direction, with clear potential for muletipronged follow-up work”.