

## Notes on revision made to manuscript

The authors would like to thank the reviewers for their constructive comments and suggestions that have helped improve the manuscript. Please see below our responses.

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### Reviewer 1

**Reviewer Comment 1.1** — “In Figure 2, the authors show what the temperature dependence of the resistance of such constricted superconductors should be. However, they do not compare them with the actual resistivity curves reported in ref. 32 and 47 on much wider samples. Now, samples with a thickness of nm and a width of mm, show rounded transitions above  $T_c$  because of the well-known contribution of short-lived Cooper pairs (This is known as Aslamazov-Larkin fluctuations). This contribution, the famous paraconductivity, is prominent up to at least 10 percent of  $T_c$ . In contrast, the effect discussed in this paper is restricted to a much narrower temperature range ( $< 0.001 T_c$ ). Just by comparing the evolution of the three theoretical curves for 0.5, 2 and 10 microns, one can see that it is unlikely to find what is seen in ref. 47 for a 1000 micron sample (such as S197 in their Table 1 and Figure 1).”

**Reply:** We acknowledge that the Aslamazov-Larkin (AL) fluctuations describe experiments at  $T \gtrsim T_{GL}$ . But we believe as long as one is sufficiently close to  $T_c$  i.e.,  $T - T_c \ll T_{GL} - T_c$ , there should be a range of temperatures where our theory is more appropriate than the AL theory. As pointed out in Ref. [26], the AL theory is a perturbative theory and cannot be trusted too close to the critical point as a matter of theoretical principle (although we acknowledge in practice it can work quite well). We have included a paragraph in section 3 pointing this out and acknowledging that how far above  $T_c$  one can get before our theory becomes a more appropriate description than the AL theory, remains an open problem. We also wish to emphasize that fig 2 in our paper was meant to indicate a change in the global resistance associated with a special constriction geometry etched in the sample. It was not meant as a comparison to experimental sheet resistivity curves reported in Refs. [32] and [47]. Given that the disorder associated with different samples varies, we believe a direct comparison to actual resistivity/ $T_c$  values, for say sample S197 in ref 47, is not so clear cut.

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### Reviewer 2

**Reviewer Comment 2.1** — “An issue of terminology: “non-Ohmic” usually means current nonlinear in voltage (and vice versa). In this paper it appears that non-Ohmic is being used to mean linear but non-local. This is potentially misleading and should be stated much more clearly, since this is quite non-standard usage. I think it would be best to not use “non-Ohmic” to mean anything other than nonlinear, thus following standard usage. Non-local is probably a clearer and less ambiguous term in this context.”

**Reply:** As requested by the referee, we have corrected our terminology of using “non-Ohmic” and replaced it with the more appropriate terminology “non-local”.

**Reviewer Comment 2.2** — “Since this paper is a discussion of systems near the Kosterlitz-Thouless transition, there should be a more careful and thorough discussion of the contributions of weakly bound vortex-antivortex pairs. The conductivity at  $k$  should, roughly, “see” weakly-bound pairs with spacings more than  $1/k$  as effectively free vortices. Since the pairs interact logarithmically with distance, this should bring in power laws with continuously variable exponents as one varies the temperature, as is standard in Kosterlitz-Thouless physics. Ref. 20 works this out for uniform currents ( $k=0$ ) as a function of the frequency. It seems that here one should do the analogous calculation, using the Kosterlitz-Thouless RG understanding of these systems, for zero frequency as a function of the wavenumber  $k$ .”

**Reply:** We agree that a more careful and thorough discussion similar to what was done in ref. 20 might reveal varying power laws away from  $T_c$  (and at large  $k$ ). However, our aim in this paper is to understand the qualitative behavior close to  $T_c$  and we feel that the complete RG calculation is not necessary for the purpose of this current manuscript which is about behavior “near the onset of superconductivity” (i.e. for  $T$  approaching  $T_c$  from above). So our arguments are ultimately about the physics extremely close to, and exactly at,  $T_c$ . As we explain in Appendix B, weakly bound vortices do not dominate  $\sigma(k)$  at small  $k$ . Given the current challenges associated with experimental sensitivity in realizing such delicate scaling, even if we were to get power laws with a variable exponent as a function of  $T_c - T$ , experimentally observing such a change would be quite hard in the regime of interest close to  $T_c$ . We have added clarifying statements on this point below Eq. (15).

The older work Ref. [21] also addresses the question of whether the vortex diffusion constant (which directly relates to the power law in  $\sigma(k)$ ) is qualitatively modified by weakly bound vortex pairs. Their conclusion agrees with ours: the vortex diffusion constant is well-defined and finite exactly at  $T_c$ . Thus, at  $T_c$ ,  $\sigma(k) \sim k^{-2}$ . We have given an alternative argument that these weakly bound vortex pairs do not change the qualitative scaling in  $\sigma(k)$  exactly at  $T_c$  in our Appendix B.