Response to Referee 2

The Referee's report:
Strengths: 1) the appearance of the 'polaronic effects' persists at finite x already in the intermediate coupling regime; 2) the polaron effective mass decreases with increasing x. 3) in the discussion, authors provide an intuitive mechanism for the latter effect in terms of the filled 'Fermi sea' that inhibits hybridization of the injected fermion at finite kT with the states comprised of a product of a free electron at k and the one-phonon at q = kT - k in case $k < k_F$ since latter states form the polaron liquid.
Weaknesses: The method has some limitations: 1) it is valid in the low-doping regime, 2) the method can not treat (with some exceptions) attractive interaction between electrons.
Report: Considering brief descriptions of both methods where authors present some obstacles they had to overcome during the development of the method, this manuscript is well written. It presents a new method to treat polaronic systems at low doping. In contrast to DMRG, where generalization to higher dimensions is highly non-trivial, this is not the case with the MA method. I strongly support the publication of this manuscript.

Our Response: We thank the Referee for their time and effort reviewing our manuscript, for acknowledging the high-quality of our work and for recommending its publication.

The Referee:
Requested Changes: If using MA it is possible to extract the effective mass at finite x more quantitatively, I suggest, that authors present a plot depicting the effective mass vs. doping for two (or more) Einstein frequencies and coupling strengths.

Our Response: We thank the Referee for this suggestion. We note that this analysis was already performed by one of us in Figures 5 and 6 of J. Phys. Mater. 5, 044002 (2022). We included these figures here for the Referee's convenience. Given that this data was already published in an open access journal, we believe that repeating it here would be inappropriate. We have added text in the manuscript to make it more clear to the readers that these results for the effective mass are available elsewhere.

Summary of Changes

1. We modified the text to make it more clear that the MA estimates of the effective masses have already been published.

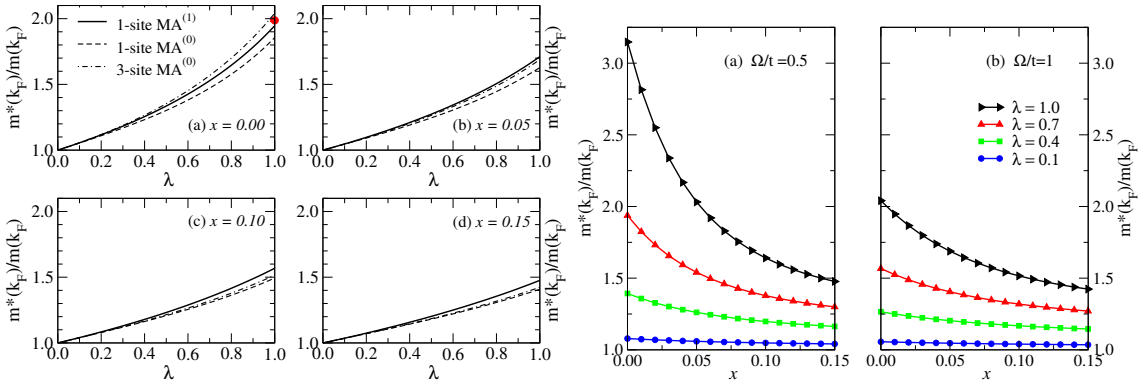

Figure 1: Renormalization of the effective mass at the Fermi energy as a function of λ for at a fixed x (left panels) and as a function of x at fixed λ (right panels). The plots are from J. Phys. Mater. 5, 044002 (2022).