Referee 1 writes "*1 - Emphasis not clear, or misleading according me. ... The Authors shows that* $\frac{d}{dt}\langle L_z^n \rangle$ *is zero for* $n = 1$*, and not zero for* $n = 2$*. I conclude that the angular momentum is conserved, but that higher momenta of it are not.*"

Answer: There appears to be a misunderstanding: for the time-indepdendent case mean-field violations of conservation laws have been known for a long time. We show the same is true for the time-dependent case, explain the origin rigorously and provide quantitative predicitions. Referee 1's conclusion that angular momentum is conserved if $\frac{d}{dt}\langle L_z \rangle = 0$, but $\frac{d}{dt}\langle L_z^n \rangle \neq 0$ for $n > 1$, is not correct in quantum mechanics. Let us explain.

1. Observables in quantum mechanics are represented by hermitian operators. Angular momentum is the operator L_z, it is not the expectation value $\langle L_z \rangle$.

2. The condition for an observable A to be conserved is $[H, A] = 0$.

3. $\frac{d}{dt}\langle L_z \rangle = 0$ follows from $[H, L_z] = 0$. It is a necessary, but not a sufficient condition for the conservation of angular momentum. See points 4, 5, 6 for further explanations.

4. It is incorrect to conclude that an observable A is conserved solely on the basis that $\frac{d}{dt}\langle A \rangle = 0$, as done by referee 1. To see this consider a single particle at rest in free space. The Hamiltonian is $H = p^2/2$. The observable x is *not* conserved: $[H, x] \neq 0$. Nevertheless $\frac{d}{dt}\langle x \rangle = 0$. Please see section 2.5 for more details.

5. $[H, L_z] = 0$ implies $\frac{d}{dt}\langle L_z^n \rangle = 0$ for all n, as shown in Eq. (9). Another proof, taken from a textbook, can be found in Appendix A. The probability distribution of measurements of L_z is only stationary if $\frac{d}{dt}\langle L_z^n \rangle = 0$ for all n. This is proven in Eq. (18). Any $\frac{d}{dt}\langle L_z^n \rangle \neq 0$ is a violation of $[H, L_z] = 0$ and is experimentally detectable by measuring L_z, see section 2.5 and the new Fig. 1.

6. Ground state mean-field solutions have been known to violate conservation laws since 1963 (Hartree-Fock) and at least since 1975 (GP mean-field). Even the methods to restore these broken symmetries have been textbook material since at least 1980. It is a well-established fact that the GP mean-field violates conservation laws. We only provide new results for the time-dependent case. See the introduction for the historical context.

7. We provide an explanation *why* satisfying conservation laws in the dynamics is only possible on the many-body level, see section 4.3 and 4.4, as well as parametric dependencies of the violations and how to fix the problem. None of this has been published before.

The above points are discussed in detail in the new version of the manuscript, including proofs, examples and references. Satisfying conservation laws in BEC dynamics requires many-body theory. We have changed the abstract to emphasize that we explain *why* this is the case.

Referee 1 writes: "*Actually I am even surprised that the error for n=2 is relatively small like the one shown in Fig.4. [now Fig.6]*"

Answer:

1. $\langle L_z \rangle = 0$ is exactly constant in the dynamics. Even in the numerics at a level of the numerical precision $< 10^{-8}$. In contrast, $\langle L_z^2 \rangle$ varies over $\pm 13\%$, a difference of at least seven(!) orders of magnitude.

2. As stated in the abstract, we chose very weak interaction strength and practically no depletion ($< 5 \times 10^{-4}$) to make the conditions as favorable as possible for the GP mean-field. Nevertheless, angular momentum conservation is substantially violated.

3. Arbitrarily strong violations: $\frac{d}{dt}\langle L_z^2 \rangle$ can take on any value between $-\infty$ and ∞ depending on the values chosen for x_0, σ_0, p_0, see Eq. (50). The violation grows linearly with the initial displacement from the center of the trap x_0, linearly with the initial momentum p_0, linearly with the GP nonlinearity parameter λ and so on. It is no problem at all to find larger violations. To illustrate this fact we have included a new Fig. 2 to show these parametric dependencies.

4. Many experiments work in the opposite limit where the kinetic energy is much smaller than the interaction energy. The violation grows with the interaction strength, see Eq. (50).

Referee 1 writes: "*Let me come to the main point: ... Actually, my question is: do it is true that $(d/dt) < p_z^n >$ is zero for $n = 1$, and zero also for n larger that 2, differently from the case considered in the paper? Do p and L_z are different for the purposes of the paper? After all, translational invariance would require exact conservation of all momenta of p (when there is no external potential), and I do not see how the argument would be different. In other words the Authors could/should make similar computations and considerations for the momentum p and the reader would like to understand why the two cases may be different (if they are). I think that such a discussion would be useful for the clarity and the substance of the paper.*"

Answer:

1. We thank the referee for suggesting to look at the conservation of linear momentum as an additional example. Our calculations are general. There is no conceptual difference between angular and linear momentum conservation. We picked violations of angular momentum conservation out of personal preference. The GP dynamics also violates momentum conservation. We have included a specific example for a dynamic violation of momentum conservation by mean-field in a new section 8.

2. Following referee 1's suggestion we now first discuss the (non-)conservation of general one-body operators A. Later we specialize to $A = L_z$ and $A = P$ and provide specific examples. However, we have kept the main focus of the paper on angular momentum.

Referee 1 writes: "*... I am also concerned about the fact that the title would lead the reader to think that angular momentum is not conserved in mean-field, while it is. " Are the (expectation values of the) higher momenta of L_z that are not conserved. So, when the Authors say "However, equations (37) and (39) are generally not zero and thus constitute explicit violations of the conservation of angular momentum in two- and three-dimensional GP theory", I would say that "constitute explicit violations of the conservation of higher momenta ($n \geq 2$) of angular momentum".*"

Answer: As discussed above, Referee 1's conclusion that angular momentum is conserved when $\frac{d}{dt}\langle L_z \rangle = 0$, but $\frac{d}{dt}\langle L_z^n \rangle = 0$ for $n > 1$ is not correct.

1. The condition for angular momentum conservation is $[H, L_z] = 0$, which implies $\frac{d}{dt}\langle L_z^n \rangle = 0$ for all n. Therefore angular momentum is not conserved in the GP mean-field dynamics.

2. The full probability distribution of the measured values of L_z involves all moments $\langle L_z^n \rangle$. If any of these is time-dependent, so is this probability distribution. See Eq. (18), section 2.4 and 2.5.

3. In quantum mechanics it is not correct to conclude that an observable is conserved only because its expectation value $\langle A \rangle$ is time-independent. It is important to realize that even for observables A that are not conserved, $[H, A] \neq 0$, one can have $\frac{d}{dt}\langle A \rangle = 0$. We have now included a single particle counter example, for details see section 2.5.

Conclusion: Referee 1 concludes (incorrectly) from $\frac{d}{dt}\langle L_z \rangle = 0$ that angular momentum is conserved by the GP mean-field dynamics, whereas for conservation of L_z in quantum mechanics all moments $\langle L_z^n \rangle$ have to be time-independent, which is not the case. Stationary mean-field violations of conservation laws have been long known. We treat the time-dependent case. We explain from first principles *why* conservation laws can only be satisfied on the many-body level in the dynamics. In order to avoid further confusion we have included the relevant background material that illustrates these points. Furthermore, we included an example for the violation of linear momentum conservation by the GP mean-field, as was asked for by Referee 1 as well as expressions for higher order momenta.