We start with Eq.(9) as the renormalized action of the O(N) vector model at scale z^*. Together with the reference action, we have

$$Z = \int \mathcal{D}\phi\mathcal{D}\mathbf{X} \Big\{ -\frac{m^2}{2}\sum_i \phi_i^2 - \frac{m^2}{2}\frac{1}{e^{2z^*}-1}\sum_{ij}\left[I - \frac{e^{2z^*}}{(e^{2z^*}-1)}\frac{1}{\mathbf{K}+\mathbf{X}}\right]_{ij}\phi_i\phi_j$$
$$-\frac{N}{2}\mathrm{tr}\log\left[(e^{2z^*}-1)(\mathbf{K}+\mathbf{X})\right] + \frac{Nm^4}{16\lambda}\sum_i X_i^2\Big\} \tag{1}$$

Here ϕ is the microscopic UV field. So this action preserves the information of the UV action and keeps track of the RG flow. Instead of working on this action, we can integrate over the microscopic field ϕ_i and get

$$Z = \int \mathcal{D}\mathbf{X} \Big\{ -\frac{N}{2}\mathrm{tr}\log\left[e^{2z^*}(\mathbf{K}+\mathbf{X}) - \frac{e^{2z^*}}{(e^{2z^*}-1)}\right] + \frac{Nm^4}{16\lambda}\sum_i X_i^2\Big\} \tag{2}$$

By inserting an auxiliary field φ, we get an effective action as

$$Z = \int \mathcal{D}\varphi\mathcal{D}\mathbf{X} \Big\{ -\frac{m^2}{2}\sum_{ij}\varphi_i\left[e^{2z^*}(\mathbf{K}+\mathbf{X}) - 1 - \frac{1}{(e^{2z^*}-1)}\right]_{ij}\varphi_j + \frac{Nm^4}{16\lambda}\sum_i X_i^2\Big\} \tag{3}$$

In the continuum limit, $\mathbf{K}_{ij}$ can be written as $\left[a - \frac{\nabla_j^2}{m^2} + 1 + \frac{1}{e^{2z^*}-1}\right]\delta(r_i - r_j)$. We can do rescaling as $\tilde{\varphi}_{\tilde{r}} = \varphi_r e^{\frac{D}{2}z^*}$ and $\tilde{\mathbf{X}}_{\tilde{r}\tilde{r}'} = \delta(\tilde{r}-\tilde{r}')X_{\tilde{r}}$ where $X_{\tilde{r}} = m^2 e^{2z^*}X_r$. At $a+1=0$, i.e. the Wilson-Fisher fixed point, this leads to a scale invariant action as

$$Z = \int \mathcal{D}\tilde{\varphi}\mathcal{D}X \Big\{ -\frac{1}{2}\int d^D\tilde{r}\tilde{\varphi}_{\tilde{r}}\left[-\tilde{\nabla}^2 + X_{\tilde{r}}\right]\tilde{\varphi}_{\tilde{r}} + \frac{N}{16\lambda}e^{(D-4)z^*}\int d^D\tilde{r}X_{\tilde{r}}^2\Big\} \tag{4}$$

When $D < 4$, the last term can be dropped. And then we obtain the familiar effective action used in previous literature. The action in Eq. (4) can also be obtained by doing the Hubbard-Stratonovich transformation to the UV action of the O(N) model.

However, Eq. (4) in terms of auxiliary field $\tilde{\varphi}$ loses information of the UV action. The relation between UV operator and IR operator can only be obtained through Eq. (1).