Referee 2 writes "*1 - the aim of the manuscript - as highlighted very strongly also in the title and abstract - is misleading to say the least ... From the title and the abstract the reader infer that the aim of the present manuscript is to show that the mean field Gross-Pitaevskii (GP) description of a Bose-Einstein condensate (BEC) is unable to properly take into account angular momentum conservation. Reading the manuscript however it turns out that the author do not mean the conservation of the angular momentum $\langle L_z\rangle$ – which indeed the author show to be well account for within GP – but of the absence of conservation of the higher order momenta of the the angular momentum distribution.*"

Answer: There appears to be a misunderstanding: for the time-independent case mean-field violations of conservation laws have been known literally for over half a century. We show the same is true for the time-dependent case, explain the origin rigorously and provide quantitative predicitions. Referee 2 states that angular momentum is conserved by the GP mean-field dynamics because the expectation value $\langle L_z\rangle$ is time-independent. This is not correct. In quantum mechanics $\frac{d}{dt}\langle L_z\rangle = 0$ is only a necessary, but not a sufficient condition for the conservation of angular momentum. Let us explain.

1. ("*... the author do not mean the conservation of the angular momentum $\langle L_z\rangle$ ....*"). Observables in quantum mechanics are represented by hermitian operators. Angular momentum is represented by the operator L_z, not by its expectation value $\langle L_z\rangle$.

2. The condition for an observable A to be conserved is $[H, A] = 0$.

3. $\frac{d}{dt}\langle L_z\rangle = 0$ follows from $[H, L_z] = 0$. It is a necessary, but not a sufficient condition for the conservation of angular momentum. See points 4, 5, 6 for further explanations.

4. It is incorrect to conclude that an observable A is conserved solely on the basis that $\frac{d}{dt}\langle A\rangle = 0$. To see this consider a single particle at rest in free space. The Hamiltonian is $H = p^2/2$. The observable x is *not* conserved: $[H, x] \neq 0$. Nevertheless $\frac{d}{dt}\langle x\rangle = 0$. Please see section 2.5 in the revised version for more details.

5. $[H, L_z] = 0$ implies $\frac{d}{dt}\langle L_z^n\rangle = 0$ for all n, as shown in Eq. (9). Another proof, taken from a textbook, can be found in the Appendix of the revised manuscript. The probability distribution of measurements of L_z is only stationary if $\frac{d}{dt}\langle L_z^n\rangle = 0$ for all n. This is proven in Eq. (18). Also this proof is taken from a textbook. Any $\frac{d}{dt}\langle L_z^n\rangle \neq 0$ is a violation of $[H, L_z] = 0$ and is experimentally detectable by measuring L_z, see section 2.5 and the new Fig. 1.

6. Ground state mean-field solutions have been known to violate conservation laws since 1963 for Hartree-Fock and at least since 1975 for GP mean-field. Methods for restoring these symmetries have been textbook material since at least 1980. There is nothing "misleading" about stating that "the

mean field Gross-Pitaevskii (GP) description of a Bose-Einstein condensate (BEC) is unable to properly take into account angular momentum conservation". It is a well-established fact. We only provide new results for the time-dependent case. See the newly rewritten introduction for details.

7. We provide an explanation *why* satisfying conservation laws in the dynamics is only possible on the many-body level, see section 4.3 and 4.4, as well as parametric dependencies of the violations and how to fix the problem. None of this has been published before.

Referee 2 writes: "*However the fact the GP approach is unable in general to properly describe fluctuations is well known. It has nothing to do with BEC, but it is true for any mean-field approach. It would be rather surprising the opposite.*"

Answer: Referee 2 is right that it is very well known that fluctuations are not accurately described by the GP mean-field or any other mean-field. However, fluctuations as such are not the topic of this work. The question we answer is what kind of approximations are capable of satisfying conservation laws such as angular and linear momentum conservation. It turns out that conservation laws can only be satisfied on the many-body level. This is not a priori clear. But we go far beyond this. Specifically, our findings are that angular momentum conservation is not only violated in the stationary case as has long been known, but that angular momentum conservation is violated by the GP time-dynamics. We provide quantitative predictions, including parametric dependencies for specific examples in a parameter regime, where one would expect the GP mean-field to provide very accurate predictions: the depletion of the condensate is less than 5×10^{-4}. But most importantly we provide an explanation of this violation based on the variational principle. We even go further by gradually restoring the symmetry through extensive many-body simulations. None of the above has been published anywhere and it is very surprising that restoring this fundamental symmetry in the time-dynamics takes a tremendous computational effort. Please also see the new example we provide to demonstrate a violation of linear momentum conservation in section 8.

Referee 2 writes: *2 - (given 1-) only the dipole mode with $L_z = 0$ is considered. And only the second order L_z^2. Further cases are needed.*

Answer:

1. A single counter example is enough to prove a hypothesis wrong. We have provided such a counter example: GP theory violates angular momentum conservation. There is no need for further cases. Nevertheless, we have included another case, see the new section 8.

2. We selected an analytically and numerically tractable case, to provide the parametric dependencies. This is why we chose the dipole mode.

3. Referee 2 asks for another example. As is clear from the theory we provide in section 4, angular momentum is nothing special. We therefore decided to provide an additional example that shows that the GP mean-field also violates momentum conservation, $[H, P] = 0$. Please see the new section 8.

4. Referee 2 asks for higher moments. While no higher orders are needed to prove the violations we went the extra mile for referee 2 and evaluated $\langle L_z^3 \rangle_{GP}$ and $\langle P^3 \rangle_{GP}$ in Appendix B. As expected they are time-dependent as well.

Referee 2 writes: *3 (given 1-) Comments on the use of Bogolyubov theory or linearised GP approach above the ground state in the linear regime to determine fluctuations - as done in literature - are completely absent.*

Answer: The referee is right that we did not use Bogolyubov theory. We have explained our findings based on the variational principle, i.e. the most fundamental level of explanation possible, see section 4. Unlike Bogolyubov theory or the linearized GP approach our approach does not rely on the assumption of a small depletion and linearization around a mean-field state. We use the full equations of motion. Obviously rigorous results are always preferable over approximate treatments when it is possible to obtain them. Thereby, we have excluded the possibility for loopholes. Had we only relied on linearized versions of the equations of motion, it could not be excluded that a violation of angular momentum conservation is merely a consequence of the linearization approximation.

Referee 2 writes: *1. the fact that the error is not very large even in the dynamics is per se an interesting result (although pointing in the opposite direction of the author's aim).*

Answer:

1. $\langle L_z \rangle = 0$ is exactly constant in the dynamics. Even in the numerics at a level of the numerical precision $< 10^{-8}$. In contrast, $\langle L_z^2 \rangle$ varies over $\pm 13\%$, a difference of at least seven(!) orders of magnitude.

2. As stated in the abstract, we chose very weak interaction strength and practically no depletion ($< 5 \times 10^{-4}$). to make the conditions as favorable as possible for the GP mean-field. Nevertheless, angular momentum conservation is heavily violated.

3. Arbitrarily strong violations: $\frac{d}{dt}\langle L_z^2 \rangle$ can take on any value between $-\infty$ and ∞ depending on the values chosen for x_0, σ_0, p_0, see Eq. (50). The violation grows linearly with the initial displacement from the center of the trap x_0, linearly with the initial momentum p_0, linearly with the GP nonlinearity parameter λ and so on. It is no problem at all to find larger violations. To illustrate this fact we have included a new Fig. 2 to show these parametric dependencies.

4. In section 8 we now provide an example for the violation of linear momentum conservation where $\langle P^2 \rangle$ grows quickly by about 600% instead of staying constant.

5. Note: many experiments work in the opposite limit where the kinetic energy is much smaller than the interaction energy. The violation grows with the interaction strength, see Eq. (50).

Conclusion: Referee 2 claims (incorrectly) that angular momentum is conserved by the GP mean-field dynamics, because $\frac{d}{dt}\langle L_z \rangle = 0$. However, as we have shown, this is not enough in quantum mechanics: all moments $\langle L_z^n \rangle$ need to be time-independent. Mean-field violations of conservation laws have been known for many decades for stationary states. Here, we treat the time-dependent case. We explain from first principles *why* conservation laws can only be satisfied on the many-body level in the dynamics. In order to avoid further confusion we have included all necessary background material that illustrates these points. Following referee 2's requests we included another example, showing the violation of momentum conservation by the GP mean-field and provide results for higher order momenta.