

We thank the referee for the careful reading of our manuscript and making several important comments. We found the comments to be extremely useful and they have helped us to improve our manuscript. We further thank the referee for mentioning several strengths of our work and finding our analysis to be systematic in approaching this class of models.

## I. THERMODYNAMIC STABILITY OF HILBERT SPACE FRAGMENTATION

Following the referee's suggestion, we have added a section on the thermodynamic stability of Hilbert space fragmentation (HSF) for the period-2 model with  $\mu = V = \omega = 20$  for  $L = 12, 16$  and  $20$ . We do not find any problems arising in the quasienergy spectrum due to folding (recalling that the quasienergy can be chosen to lie in the range  $[-\omega/2, \omega/2]$ ) for the above parameter values and system sizes. One can argue that the many-body bandwidth is significantly suppressed due to the dynamical localization at these parameter values, and therefore, the quasienergy spectrum evades the folding problem at these special points. We would also like to mention that investigating the thermodynamic stability of HSF for smaller values of  $\mu = V = \omega$  would not be helpful in our opinion since the first-order effective Hamiltonian (which gives HSF) breaks down with decreasing values of  $\mu, V$ , and  $\omega$ , and the higher order corrections (which are likely to destroy HSF) will become more and more significant. Therefore, we have examined the thermodynamic stability of HSF with values of  $\mu = V = \omega$  where the first-order effective Hamiltonian is a good approximation to the exact effective Floquet Hamiltonian. We hope that this new section in the revised manuscript will satisfactorily address the referee's concern about this issue.

## II. REQUESTED CHANGES

1. The set of references cited for dynamical localization at the end of p. 2 [74-79] seems to be a mistake and probably should be [58-64] (cf. top of the page).

We thank the referee for pointing out this typo and we have now corrected it.

2. Can the authors check if the  $\sum_n$  in Eq. (13) needs to exclude  $m = n$ ? Presumably the matrix element of  $H_1$  vanishes in this case, but it's perhaps better to be explicit.

We thank the referee for this question. The  $\sum_n$  in Eq. (13) does include  $m = n$  in general, and one can derive this result through the usual perturbation theory. However, as the referee has correctly pointed out, the diagonal matrix elements, i.e.,  $\langle m|H_1|m\rangle$  turn out to be zero for our model.

3. The figures appear to be rasterized in low resolution. To reach publication quality, the figures should be vectorized. To limit file sizes, in some cases only the panel content should be rasterized (e.g. Fig 4a, 4b etc).

We thank the referee for the comment. However, we have decided to keep the figures as they are for now.

4. In several occasions, the limits  $\mu \gg J$  and similar are typeset with a double  $>$  sign. It's better to replace this by  $\gg$ . (p.6, 8, p.11, 14, 15, etc.)

We thank the referee for this comment. We have now used a  $\gg$  sign.

5. Fig 2 shows the crossover of the correlation function  $\delta C_n$  as a function of the number of cycles. Can the authors show the derived result for the crossover  $n_c \sim 1/|\epsilon|^2$  in the figure?

We thank the referee for pointing this out. We have now added a plot (Fig. 2 (e)) showing how the crossover scale  $n_c$  diverges as one approaches  $\omega_c$  from the  $\omega > \omega_c$  side by varying  $|\epsilon|$ . The plot clearly shows that  $n_c \sim 1/|\epsilon|^2$ , and therefore confirms our analytically derived result.

6. The authors show in Fig. 5 the entanglement entropy for exact eigenstates of the Floquet operator and for eigenstates of the 1st order FPT Hamiltonian and say that the quasienergies agree "quite well". It would be interesting to show this comparison directly. One way to do this would be to order both spectra by the phase angle (quasienergy) and then plot  $E_{exact}$  vs.  $E_{FPT}$ . A straight line would indicate an exact match, and deviations quantify the agreement.

We thank the referee for making this very helpful suggestion. In the revised manuscript, we have included a plot of  $E_{exact}$  vs  $E_{FPT}$  and have fitted it numerically to quantify the agreement of the first-order Floquet perturbation theory with the exact numerically obtained results.

7. Is the Fig. 5b showing the spectrum of Eq. (36) or Eq. (42)? It would be helpful to indicate this in the figure caption.

We thank the referee for this question. Fig. (5b) has been obtained from Eq. (42). We have indicated it in the figure caption of the updated version of the manuscript.

8. Fig 5a) and to a larger extent 19b) show states at zero quasienergy with an excess entanglement entropy compared to the rest of the spectrum. This is reminiscent of what is seen in the PXP model, where the origin is a large degenerate subspace at zero energy and the numerically obtained eigenvectors are an arbitrary orthonormal basis of this subspace? If so, there is no physical content to these points and it would be good to check if there is such a degeneracy.

We thank the referee for this question. We do not observe any large degenerate subspaces at or close to zero energy. Rather, we find that the energy spectrum only has a two-fold degeneracy (coming from the zero momentum sector) due to a particle-hole symmetry at half-filling. Therefore, as pointed out by the referee, the numerically obtained eigenvectors can be arbitrary orthonormal combinations of such pairs of degenerate states. This is possibly causing the excess entanglement entropy compared to the rest of the spectrum.

### III. LIST OF CHANGES MADE IN THE REVISED MANUSCRIPT

We have shown the major changes in blue in the revised manuscript.

1. We have added two sections, namely, the thermodynamic stability of Hilbert space fragmentation in this class of models (Sec. 5) and the experimental accessibility (Sec. 6).
2. We have added the derivation of the third-order effective Hamiltonian at the dynamical localization point in Appendix C. This is relevant for answering a question asked by the first referee.
3. We have removed Fig. 1 (b). However we have retained Fig. 1 (c), which is important for seeing the variation of the bandwidth due to the third-order corrections which become larger when the value of  $\mu$  is decreased.
4. We have added two figures (Figs. 23 (a) and (b)) showing the variation of the Loschmidt echo with time for the resonant case of the period-2 model at a dynamical localization point for two sets of parameter values. We have numerically fitted the envelop of the Loschmidt echo varying with time to extract the decay rate of the envelop and have discussed how the decay rate is related to the parameter values used.
5. We have added a figure (Fig. 2 (e)) showing how the crossover scale  $n_c$  diverges as one approaches the critical frequency  $\omega_c$  from the  $\omega > \omega_c$  side. This further confirms our analytically derived result.
6. We have added a figure (Fig. 5 (c)) showing a plot of  $E_{exact}$  vs  $E_{FPT}$  for the resonant case of the period-2 model at a dynamical localization point. We have fitted the plot to quantify the agreement of the first-order Floquet perturbation theory with the numerically obtained results.
7. We have mentioned certain symmetries of our effective Floquet Hamiltonian obtained for the period-2 model at resonance and at a dynamical localization point to contrast this kind of Hilbert space fragmentation (HSF) with the models showing HSF which were known earlier.
8. We have corrected some typos pointed out by the referees.