

Authors' response to Referee #1

The authors studies non-Hermitian degeneracies in non-Hermitian systems subjected to symmetries. In particular, the authors address defective, non-defective and ordinary Hermitian degeneracies. Of great importance is the recipe to distinguish the three types of degeneracies in non-Hermitian setups, which is of importance towards the detecting of exceptional points and their potential applications. Furthermore, I would like to highlight that the manuscript is well-written and the messages clearly exposed. For these reasons I recommend its acceptance in Sci-Post after addressing the minor comments listed below, which might be useful to further improve the manuscript:

Authors' response:

We are pleased that the Referee finds our work **important** and **well-written**. We are also happy that the Referee recommended our manuscript for publication.

(1) In page 10 below equation 29, in the sentence starting with "This can be seen from Fig. 4(b)..". I have the feeling that it should be Fig.4(c).

Authors' response: We thank the Referee for spotting this typo, which is now corrected in the revised manuscript.

(2) I would recommend the authors to add additional references, highlighting the immense efforts of several groups. For instance, see following suggestions:

Authors' response: We are grateful that the Referee suggests further literature that could be of relevance to our work. Below, we list which suggestions are included in the bibliography, and explain why we excluded some.

(a) Together with Refs. [1-5], the authors could incorporate the first non-Hermitian studies on a consented matter matter junction: JETP Lett. 94, 693 (2012); Scientific Reports 6, 21427 (2016).

(b) Together with Refs. [6-19]: Phys. Rev. B 99, 165145 (2019); Phys. Rev. Research 4, L012006 (2022).

(c) Together with Refs.[28-38], I recommend adding: Phys. Rev. B 97, 014512 (2018); Phys. Rev. B 107, 104515 (2023); Phys. Rev. B 107, 115146 (2023); Proc. Int. Conf. on Strongly Correlated Electron Systems (SCES2019) (Physical Society of Japan, 2019) Chap. 30, p. 011098.

Authors' response: We have added references (a) and (b) in the manuscript according to the Referee's suggestion.

Of the references in (c), we have only added the last one. The reason for this is that the other references do not discuss exceptional points and their relation to symmetries in a general enough manner but rather treat the physical consequences of such degeneracies. That being said, these references are, of course, still interesting, but not relevant when discussing how symmetries generally affect the codimension of exceptional points.

In section 4, the authors discuss about identifying exceptional points in experiments. Even though the authors address some relevant systems, I believe their manuscript could have a bigger impact if they also discuss non-Hermitian systems emerging from material junctions. The material junctions are very relevant in transport experiments, see e.g., Nat. Rev. Mater. 6, 944 (2021), and represent a natural

platform for non-Hermitian physics. Examples of non-Hermitian physics in junctions have been initially reported in JETP Lett. 94, 693 (2012); Scientific Reports 6, 21427 (2016). Later, more interesting studies were reported which further support the importance of non-Hermitian physics in material junctions, see e.g., Phys. Rev. B 87, 235421 (2013).; Phys. Rev. Research 1, 012003(R) (2019); Phys. Rev. B 103, 235438 (2021); J. Phys.: Condens. Matter 35, 254002 (2023).

Authors' response:

We thank the Referee for bringing these references to our attention. We agree with the Referee regarding the plausible realization of our findings in heterostructures. However, we emphasize that our main goal for Sec.4 was establishing the idea that our proposed distinctions of different degeneracies in non-Hermitian systems have experimental indications. To the best of our knowledge, the emergence of non-defective degeneracies and the possibility of capturing both defective and non-defective degeneracies in the energy spectra of heterostructures has yet to be reported. Despite this fact and to bring this idea to the attention of readers, we added a paragraph

“In addition to the setup mentioned above, numerous studies on heterostructures report the occurrence of exceptional points in these systems. However, to the best of our knowledge, no record of non-defective degeneracies is reported in these systems [6, 67-69]. Nevertheless, further engineering of the structure of these systems to maintain the symmetries discussed here may allow for the emergence of non-defective degeneracies and the experimental realization of our findings in these setups.”

to present heterostructures as other plausible platforms to detect various non-Hermitian degeneracies.

Summary of changes:

- We have added

“The microwave experiments with a metallic mesh 3D photonic crystals have also realized \mathcal{PT} -symmetric models [67]. Here, defective EPs form chains, and their intersection of these EP lines represent non-defective EPs. These non-defective degeneracies in these particular experiments are sometimes protected by additional mirror symmetries; see Refs. [68,69] for details.”,

”Aside from realizing NH degeneracies, the occurrence of these degeneracies in the spectrum may give rise to exotic responses. The NH anomalous currents observed in odd spatial dimensions exemplify these interesting responses. It has been shown that NH (non)interacting systems with ONPs, when coupled to gauge fields, e.g., electromagnetic fields, exhibit anomalous currents different from their Hermitian analog [5,70,71]. For instance, the NH chiral magnetic effect, in contrast to its Hermitian counterpart, may find room to emerge in equilibrium in \mathcal{PT} -symmetric systems [70].”

and

“In addition to the setup mentioned above, numerous studies on heterostructures report the occurrence of exceptional points in these systems. However, to the best of our knowledge, no record of non-defective degeneracies is reported in these systems [6, 67-69].

in section 4.

- Figures 2 and 3 in the previous version of the manuscript (Figures 3 and 4 in the updated manuscript) have been updated.
- The symmetry constraint defining TRS^\dagger has been corrected.
- A schematic figure explaining the differences between the various eigenvalue degeneracies has been added as a new Figure 1.
- A list including formal definitions of the various degeneracies has been added.
- Corrections of minor typos throughout the manuscript.
- Corrections of sign mistakes in Eqs. (17), (18), (21) and (22).
- Change of limits in Eq. (15).
- Addition of discussion on topological invariants for non-defective EPs.
- Expansion of the reference list as mentioned in the above answer.