
Second Response to Anonymous Report 3

The authors have addressed of all my main comments. Concerning point 5, to clarify, what I meant is the correlation between the spatial distribution of the SC gap Δ and the disorder potential, for a fixed realization of the disorder. One expects that, as the authors claim, that the vortex cores are pinned to areas with strongest disorder. Figure R4 seems to suggest that this is indeed the case, at least for stronger disorder, but it is not clear from data presented in the manuscript. A plot of V(r) alongside the plots of Δ(r) already in the text would address my question. This seems especially important given the discussion by the authors of the need for a self-consistent BdG calculation, since they state that the disorder in Δ(r) does not follow the same distribution as the disorder in V(r).

Our response:
Thanks for the clarification. We now fully understand the referee's comment. We have included a new paragraph (page 9, when we comment about Fig. 4) in the updated version of the manuscript where we explain explicitly why the order parameters spatial distribution can follow that of the disordered potential only in very specific circumstances such as the very weak disordered region. However, we feel it may be distracting to add an explicit comparison between the two when the paper focus is about the interplay of disorder and magnetic field. From our point of view, it is really well established (see below) that Δ(r) have the intricate spatial structures while V(r) do not have. It is indeed quite explicit from Fig. 4 for no flux, left column. While our random potential has a flat box distribution, it is clear that, except in the very weak disorder region (V=0.5) the order parameter has a much more intricate spatial structure.

As a further confirmation, in Figure. R1 below, we depict the spatial distribution of Δ(r) and V(r). For convenience, we plot |Δ(r)| and |V(r)|, since both large positive and negative potentials are detrimental of superconductivity. Clearly, |V(r)| is distributed completely randomly without any specific structures. However, due to the quantum interference of the wave functions, together with the self consistent condition, |Δ(r)| has a much richer spatial structure. Indeed, there is ample experimental [Nano letters 20.7 (2020): 5111-5118, Nature Physics 15.9 (2019): 904 and numerical evidence Physical Review B 65.1 (2001): 014501, Physical Review B 101.10 (2020): 104509] supporting this view.

To be more specific, in the region marked by the red circles, the potential contains both small and large potentials, however, the order parameter is always suppressed. In the superconducting islands, there are also many sites with relatively large potentials. It is clearly that Δ(r) and V(r) do not follow the same distribution or are close in any evident way. The quantum interference makes the superconductor distribution more

complicated. We hope that this issue is fully clarified now.

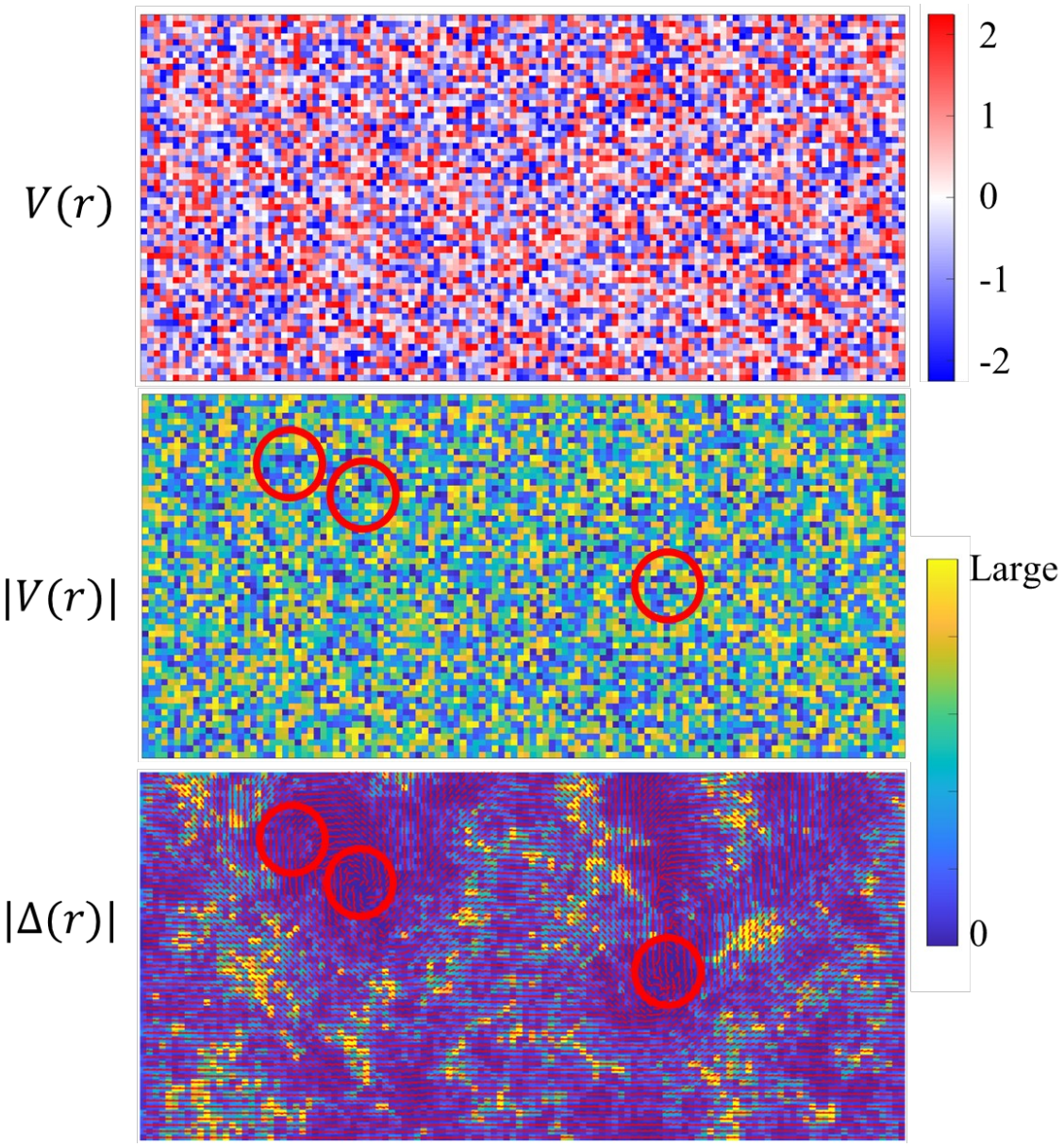

Figure. R1 The spatial distribution of disorder V(r), and it's absolute value |V(r)|, and the superconducting order parameter amplitude |Δ(r)| with its phase (red arrow). The system size is 60*120, the coupling constant |U| = 1, the charge density <n>=0.875, the magnetic flux is 2, and the disorder is distributed between [-2.25, 2.25]. The spatial distribution of disorder is completely different from that of order parameter. Note that for this disorder strength, the spatial distribution of the order parameter is rather insensitive to such a weak flux.

Additionally, I would suggest another proof-reading of the text as many grammatical errors remain. For example, on page 15: "where it is observed a clear deformation" should instead read "where a clear deformation is observed." Another common mistake is the use of "the vortices position," which should either read "the vortex position" or "the position of vortices." Once these are fixed, I believe the manuscript

will be acceptable for publication.

Our response:
We thank the referee for this suggestion. We have carefully proofread the manuscript again. Hope that we have fixed all the typos and grammar errors.