
Responses to Referee 1:

We thank the referee for the careful reading of our manuscript and the positive comments that “I find the paper interesting”, “I particularly find the momentum space construction and the invariant calculation interesting” and “it might be useful for the community of researchers working on topological phases away from crystalline settings.” We thank his/her recommendation of publication of our work in SciPost Physics. We also appreciate his/her constructive suggestions/questions which are invaluable in improving our paper. The following contains our detailed response to specific points.

Strengths: The paper shows that higher order phases which are protected by rotational symmetries - which cannot be achieved in crystals - and were shown to be realised in quasi-crystals can also be realised in amorphous systems.

- 1) To that effect the authors show that while the boundary modes get realised in any typical system; to well-define a topological invariant they need to make some equivalent systems where the lattice is deliberately made rotationally invariant and a band structure can be made by artificially repeating the pattern. They show that the region where the Pfaffian of this system is non trivial - is consistent with non-trivial phase of the system and associated gap closings.
- 2) They also calculate the real space quadrupolar moment in the system by deforming the lattice into a square patch which is consistent with these transition values.
- 3) To further investigate the low energy modes near half-filling they look at typical level spacing ratio and find that transitions between topological and trivial phases are characterised by change in ratio from GOE ensemble to poissonian which is interesting.

Reply: We thank the referee for the nice summary of our results.

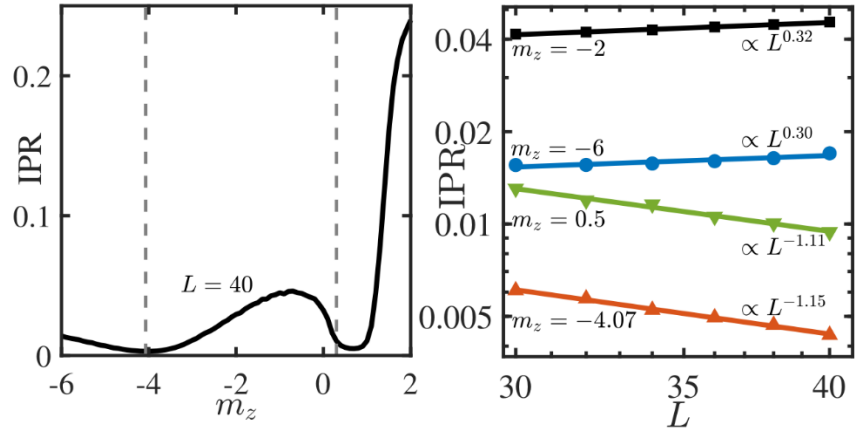
Report: I find the paper interesting and recommend publication. I particularly find the momentum space construction and the invariant calculation interesting and it might be useful for the community of researchers working on topological phases away from crystalline settings.

Reply: We thank the referee for the positive evaluation of our work and recommendation of our work for publication in SciPost Physics.

Comment 1: I was wondering if the IPR of the states close to Fermi energy show any anomalous scaling near the transitions as a function of m_z . It will be useful if the authors can comment on that.

Reply: We thank the referee for the nice suggestion. We have followed the suggestion to calculate the IPR of the states near zero energy with respect to m_z and plot it in the following figure. We see clearly that the IPR is small at the transition point and the intermediate critical region and is large in the Anderson localized regions, which is consistent with the results of the LSR. We also plot the scaling of the IPR with system sizes and find that at the transition point

and the intermediate critical region, the IPR exhibits a power law decrease with system sizes, providing further evidence that the bulk states near zero energy are delocalized in these regions. However, in other regions, the IPR exhibits a slight increase with system sizes, suggesting that the states are localized. Note that the increase behavior arises from finite-size effects. In other words, the positive slope declines with system sizes so that it approaches zero in the thermodynamic limit.



We have added two figures in Fig. 3 and the corresponding discussion on the IPR in the revised manuscript.