Response to referees

Cheng-Ju Lin[1] and Liujun Zou[1]
[1] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

Contents

We thank Editor for handling our manuscript and Referees for their reports, which are helpful for us to improve our paper. Below we address the comments and questions of the referees. We also attach a summary of changes at the end of the response. Because of both referees' suggestions, we agree to transfer our paper to Scipost Core.

I. Response to referee A

Report: *The paper is interesting but I think SciPost core fits more for the publication.*

The definition of EESBO should be highlight. Maybe talk about the lack of a basis that enable to write the ground states in a product form. Maybe also put an example of not EESBO and long range like the GHZ state from the 2-fold degenerate ground state of the Ising model.

Response: We thank Referee for thinking that our paper is interesting, clear and simple, and for the comments, questions and suggestions. We have highlighted the definition of EESBO (and the "classical" order) to make them more visible, and also mentioned the ferromagnetic states and the VBS states as the example of the "classcal" order.

Report:*I have some questions about the mechanism of EESBO. The paper considers SPT phases that are not conventional but I think this is not required. Let us consider an ordinary $G_0 = Z_2 \times (Z_n \times Z_n)$ and go to $G = Z_n \times Z_n$; breaking the Z_2 symmetry that could be internal and not a lattice symmetry. With the regular SPT classification the 2 ground states could be in n different SPT phases, $n-1$ of them non-trivial and therefore not able to be written as a product state because of the entanglement given by the projective representation. Why are not those EESBO? If they are, why are they not considered?*

I guess that the family of states can be generalized to break T to T^m and have m-degenerate ground states.

Response: As far as we understand, the main question is about the difference between EESBO and the coexistence of spontaneous symmetry breaking (SSB) and nontrivial SPT. This question is answered and highlighted in the last paragraph of Sec. 4. Just to reiterate, usually in the phenomenon of coexistence of SSB and SPT, the ground states do not have to be a nontrivial SPT, although they can be. If the ground states are in the trivial SPT phase, then the usual assumption is that they can be represented by local product states. However, in our definition of EESBO, it is required that no symmetry-breaking ground state can be represented by a local product state.

Specifically to the example considered by Referee, it is indeed possible that the $G = Z_n \times Z_n$-symmetric state is nontrivial, so we have a coexistence of SSB and SPT. However, the given input is not enough to rule out the possibility that the $G = Z_n \times Z_n$-symmetric state is trivial, and which scenario of the ground states we get really depends on the details of the Hamiltonian. However, if due to the incompatibility between the product state description and the $G_0 \to G$ relation, the $G = Z_n \times Z_n$-symmetric state can never be trivial, then it is an EESBO. In other words, our contribution is the "no-go aspect": for an EESBO, the $G = Z_n \times Z_n$-symmetric state can never be a local product state.

To further highlight this difference, we have added a paragraph to clarify this point at the end of Sec. 2.

Report: *Most readers won't be familiar with these unconventional SPT phases. I am familiar with the standard understanding of projective representations at the bonds giving rise to standard SPT phases (also with MPSs) and the RGFP cartoon picture of entangled pairs between sites. Is there an easy characterization or cartoon picture of these unconventional SPT phases? I think it would help a lot if the paper describes them.*

Response: We thank Referee for the question and suggestion. However, as we mention in the Discussion section, it is beyond the scope of this paper to classify different EESBOs, and so far we do not have a unified picture for all EESBOs. For the unconventional SPT phases, they arise when some degrees of freedom on the lattice transform in projective representations of the symmetry group, while in the conventional study they are assumed to transform in linear representations. We provide cartoon pictures for the particular examples discussed in this paper, such as figure 2.

II. Response to referee B

Report: *The manuscript "Entanglement-Enabled Symmetry-Breaking Order" seems interesting to me. As I understand it, the strategy is to first specify a symmetry of a full hilbert space (or define a symmetry), then consider how this symmetry might break. If the broken symmetry can never be a local product state, then it is declared to be a EESBO.*

What is not clear to me is whether this manner of classifying broken symmetries is going to turn out to be useful or not.

Response: We thank Referee for thinking that our paper is interesting. One of the main goals of this paper is to point out the possibility of the physical phenomenon of EESBO. As we discuss in the Introduction, this is conceptually important, because if it is always assumed that all states with spontaneously broken symmetries can be represented by a local product state, some physical phenomena may be missed. One example is in the systematic understanding of the properties of magnons.

On the other hand, in this paper we do not aim to classify all EESBOs or all states with spontaneously broken symmetries. As mentioned in the Discussion section, this is one of the open problems.

Report: *There appears to be no requirement that the state in question is gapped, or even that it is the ground state of any Hamiltonian. This work is simply making statements about wavefunctions with some symmetry that live within a Hilbert space with some symmetry. If this is so, is this a feature or a bug? Does it suggest that the classification may end up classifying many things that are not phases of matter in any sense?*

Response: We thank Referee for these questions. We require that the states of interest should be ground states of some local Hamiltonian. We have added a sentence in Sec. 2 to clarify this point. However, we do not require the Hamiltonian to be gapped. For example, we allow the possibility of the gaplessness due to the Goldstone modes from spontaneous symmetry breaking.

Report: *Overall I nonetheless think the paper is interesting and should be published. I'm not sure there will be overwhelming interest, so I suggest SciPost core rather than the flagship SciPost.*

To give some suggestions, I do have to say that the paper was extremely hard for me to read. While there is a small community who is very familiar with these types of arguments, I fear that most even well-educated condensed matter theorists will find much of the arguments to be very obtuse.

The examples seem needlessly difficult to think about. For example, section 5.2 seems the simplest example (at least to me) because one can take a limit fo $a = b = c = 1$ *and then the wavefunction is super-easy to describe and you can just look at it and see what is going on. Trying to do it in generality makes it completely impossible to understand. Similarly, the entire example in section 4 is insanely arcane.*

Even in section 5, the authors insist on doing a 3d example, presumably to evade Mermin-Wagner. But there is no point in this. Since we didn't ever specify a Hamiltonian, why not just assume it is a long-ranged Hamiltonian, so that Mermin-Wagner does not apply. (Indeed, the Hamiltonian doesn't matter anyway!) Then you can just talk about spin chains (Am I wrong about this?).

I would recommend that before publication, the authors try very hard to simplify much the discussion, clarify the writing, and put all the simple examples up front. Yes, I know this is hard to do, but it really will pay off in the end. If you write a paper that only a tiny fraction of the community can bother to understand, then it won't have much impact.

Response: We sincerely thank Referee for these suggestions.

The purpose of the example in Sec. 4 is to demonstrate the existence of EESBO via an exaclty solvable Hamiltonian. A potential downside of an exactly solvable example is that the technicality could make it hard to comprehend, which is indeed an unfortunate side effect. On the other hand, for the examples in Sec. 5, we only have wavefunctions, but not Hamiltonians. Therefore, for the purpose to firmly establish that the phenomenon of EESBO is possible, we opt for putting the examples in Sec. 4 before those in Sec. 5. In fact, it is indeed an interesting future problem to find explicit models that realize the EESBOs in Sec. 5, as we mention in the Discussion section. Moreover, the examples in Sec. 5 are actually in some sense more subtle than the ones in Sec. 4, as discussed at the end of Sec. 5.2.

We specifically thank Referee for suggesting that we should discuss the case with $a = b = c = 1$, for the example in Sec. 5. There the goal is to prove that the state represented by that matrix product state is indeed short-range entangled and has a Z_2 symmetry (but no other symmetry), which is shown in Appendix C1. As one can see from Appendix C1, this discussion is in fact rather technical, and we think it is more appropriate to leave it to the Appendix. Specifying to the case with $a = b = c = 1$ in this discussion does brings in some simplification, but given it is already in the Appendix and the general case is not too complicated than this special case, we think it may be appropriate to discuss the general case directly. However, to make the presentation in the main text easier to understand, we have added a sentence about the case with $a = b = c = 0$, which does have an unwanted remaining $U(1)$ symmetry. And some intuition for the generic nonzero a,b,c wavefunction can be derived from it.

From the answer to the previous question, we do require the particular symmetry breaking order to be realized in the ground states of some local Hamiltonian.

Report: *Also note, the paragraph where the EESBO is defined in section 2 gives the definition almost as a side comment. One can read the paragraph and not even realize that there has just been a definition made. Please state it clearly and precisely, not so casually, so people know what you are talking about! And flag it clearly "DEFINITION: Given X,Y, Z, we say that a wavefunction is EESBO if P, Q, R"*

Response: We thank Referee for the suggestion. We have highlighted the definition of EESBO as suggested.

III. Summary of changes

Along with various minor changes, here is a summary of the major changes.

1. We have highlighted the defintion of EESBO.
2. We have added a sentence in Sec. 2 to emphasize that we are considering ground states of local Hamiltonians.
3. We have added a sentence at the end of Sec. 2 to emphasize the difference between entanglement-enabled symmetry-breaking orders and the usual phenomenon of coexistence of spontaneous symmetry breaking and nontrivial topological phases.
4. We have added a sentence in Sec. 5 to provide further insights to the example there.