Response to Reviewer #1

We thank the reviewer for their careful reading and positive review of our work.

STRENGTHS

> This is a careful classical Monte-Carlo study of the classical anisotropic Heisenberg model (XXZ model) with an effective barrier potential term and a random field mimicking disorder.

We thank the referee for the appreciation of our work.

WEAKNESSES

> The authors spend quite some time to convience the reader that the model they consider should capture the phase diagram of superconducting cuprates and even put this in the title. There is a series of assumptions to make such a claim and the model described above cannot explain the physics of layered cuprates. In my opinion this model is an oversimplification of the physics of the cuprates.

We agree that the model is phenomenological, based on assumptions and simplifications. However, we respectfully disagree that the model cannot describe the physics of cuprates. Below, we further motivate our statement by punctually addressing the issues raised by the referee.

REPORT

> - I think the authors should clearly put in the title the model they consider, namely XXZ model. I think the results of this paper are beyond the interest of the high-Tc community and one should catch those by appropriately changing title. Discussing the connection to the cuprates they have to make very clear the limitation of the model.

We thank the referee for this comment and added XXZ to the title. We also followed the recommendation of expanding the discussion on the limitations of the model. Furthermore, in view also of the recommendation of Reviewer 2, we added "phenomenological" to the title.

> - in my humble opinion the model the authors consider has nothing to do with the physics of cuprates. All these mapping from the negative to attractive Hubbard model and then using pseudospins can be done but after the mapping performed there is no space for the true magnetic fluctuation dominated physics. Whereas in cuprates CO and superconductivity (which is by the way d-wave while in their case it is s-wave) compete under the umbrella of the strong magnetic fluctuations. So this means there is a strong player (bosonic 'true spin fields') which the authors completely ignore and they will change the phase diagram and the details of the competition between CO and superconductivity.

We respectfully disagree with the referee. We are completely agnostic regarding the mechanism giving rise to both superconductivity (SC) and charge order (CO). Our only assumption is that there is a strong local attraction, which can result in either a charge-ordered or superconducting state. All the physics of the bosonic mediator is implicitly considered in the attractive interaction. Our model is also agnostic regarding the pairing symmetry. More precisely, we motivated the model starting from a negative-U Hubbard model, but similar models can be derived starting from d-wave paring. Notice that the 2D superconducting transition in the case of d-wave symmetry still belongs to the BKT (XY) universality class (see e.g. V.J. Emery and S.A. Kievelson, Nature **374** 1995). On the other hand, we recognize, as Referee 1 pointed out, that the phenomenological character of our model was not sufficiently emphasized. Therefore, we added a paragraph at the end of the introduction where we discuss the phenomenological character of the model, the applicability to a larger class of situations (including *d*-wave superconductivity), and at the same time the limitations in all these situations.

REQUESTED CHANGES

- Significance: title need to be changed appropriately, the authors study XXZ model with disorder

We have changed the title accordingly.

- Validity: the discrepancy between the model they consider (CO and s-wave superconductivity) should be contrasted with the actual cuprate physics (CO and d-wave in the presence of strong magnetic fluctuations)

We have now discussed this issue explicitly.

Validity: Ok Significance: Ok Originality: Ok Clarity: Ok Formatting: Good Grammar: Good