Response to Referee 2

Feb. 2024

1. We agree that the equations may be covariantised to any coordinate system using the same methods as already employed in Sec. 3.3. However, we explicitly need these expressions in spherical coordinates in Sec. 4. Therefore, we chose to consider only this explicit coordinate transformation. For concreteness, we prefer to keep that section in its current form. In the introduction of Sec. 3.3, we already mention that it is possible to work in arbitrary coordinate systems. To further drive this point home, we have added the sentence "Although the methods we use below can be generalised to arbitrary coordinates, we will need the explicit expressions in spherical coordinates in Section 4." towards the end of the introductory paragraph.

   We do not quite agree with the second part of the referee's remark: conventions vary, and we find it enlightening to emphasise that the form of the function Ψ changes when written in a different set of coordinates. As such, writing $\Psi(x) = \Psi(x')$ would be, in our opinion, very confusing.

2. We have added that Ψ "is a scalar under spatial reparameterisations" above Eq. (A.23).

   We thank the referee for pointing out this alternative approach to obtaining (A.43), and we have added a small paragraph describing this approach at the end of Sec. A.2.2, including a footnote thanking the referee.

3. - We have replaced "normalised wave function" with "wave functions with the standard inner product of nonrelativistic Quantum Mechanics".
   - This has become standard terminology. To address this, we have put "extrinsic curvature" in quotes and added a footnote commenting on the context of the terminology.

4. We have rearranged the references according to the referee's wishes.

5. - It is true that calling these objects inverses is slightly sloppy language. However, this is standard terminology; in particular, we want to emphasise (as we indeed do below) that Eq. (2.3) describes a Lorentzian

structure, and that a precise meaning can be given to the inverses when expressing all objects in (2.2) in terms of vielbeins. The same is true for the expansion: at LO, we can write $h_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu$, where a is a tangent space index and e^a_μ is a spatial vielbein. The complex (τ_μ, e^a_μ) makes up a coframe, with a corresponding frame given by $(-v^\mu, e^\mu_a)$, where $h^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab}$. The frame and the coframe are each others' inverse in the standard sense.

- We have changed "structure" to "causal structure" as requested by the referee.
- We have modified the caption of Fig. 1. While sometimes called a "velocity", v^μ is really the inverse timelike vielbein (and part of the frame, as explained above). To avoid confusion, we no longer refer to v^μ as a "velocity". In addition, we have changed "orthogonal" to "pointing away from".
- We have added "symmetric".
- Referring to vector fields $X \in \mathscr{X}(M)$ on a manifold M simply as vectors is a widespread convention, which we choose to follow. Just as we refer to tensor fields simply as "tensors", we prefer to keep referring to vector fields as vectors to avoid verbosity.
- We do not claim to prove in a mathematical sense that the inner product arises in the way we describe. Instead, we show how (as in "demonstrate the steps that leads to") the inner product that we use is related to the KG inner product. We believe this to be clear from the context, and hence we prefer to keep the original phrasing.

6. We have added the references suggested by the referee.

7. - We have added "beyond the Newtonian limit" in the spirit of the referee's suggestion.
   - The paragraph in question is intended as a schematic description of the minimal coupling prescription, keeping it as simple as possible in this opening section. We are of the opinion that replacing "derivatives" with "partial derivatives in inertial coordinates" as suggested by the referee, though technically more precise, does not fit with the spirit of the paragraph.
   - We do not agree with this statement. The schrödinger equation (1.1) describes the time evolution of any wave function.
   - We have replaced "forces" by "effects".
   - We have changed the equation according to the referee's wishes.
   - We have changed the sentence as per the referee's request.
   - We have modified the sentences as requested by the referee.
   - We have added "for suitable matter" as requested.

- We have added “as a scalar”.
- We have added a reference to Eq. (3.7), which shows the decomposition of the KG field. This should clarify the derivation.
- The field m_μ can be viewed as a gauge field, so we prefer to keep referring to $(dm)_{\mu\nu}$ as a field strength. One of the points of our paper is that m_μ precisely appears in covariant derivatives in the same way a “normal” gauge field would, see, e.g., Eq. (3.16).
- Yes, we do make factors of c explicit by plugging in Eq. (4.5).
- We have added “(up to non-minimal terms)” as requested.

8.
   - Exactly how the covariant derivatives are related to the expansion of (representations of) the Poincaré algebra is not understood, so we prefer not to mention it here.
   - We wrote “to take the standard $L^2(\mathbb{R}^d)$” as requested.
   - Since we are in curved space, this is not true and we would have to introduce spacetime covariant derivatives in the expression written by the referee.
   - We have added “as indeed they must since the KG theory is Hermitian”.

9. We thank the referee for pointing out these typos.

10. In view of the many changes already made, we decided to leave these suggestions alone.