We thank the reviewer for taking the time to analyze our work and for having recognized the importance of our findings. Here we provide a point-by-point response. Each reply is highlighted in red and follows the Reviewer's comment.

*This paper studies the interesting topic of topological phases in 2D Photonic Crystals. Both the architecture of the crystals and the methodology have been previously introduced in the literature (Ref 37). The paper rather focuses on the system's behavior regarding a change of some geometrical aspects. Specifically, it measures , through numerical means, the band gap variation as a function of the ratio of the size of the Ge stand to the lattice cell, as demonstrated in Figure 1. One of the main results of the papers is the claim of the presence of a topological phase transition.*

*Despite the paper's elegant presentation, I have several concerns:*

*1. The authors state that compressed and expanded PC belong to distinct topological phases, with the bands' parity serving as the topological invariant. However, the arguments presented are not convincing. The two configurations are related by a translation (a/2,a/2) and one can notice from figure 2 that wavefunctions of the electromagnetic field could simply be related by the same translation. both for the lower and upper bands. So, the change of parity seems trivially a result of the change of reference. It would be helpful to clarify this.*

We thank the reviewer for pointing out a possible weakness in our presentation. The topological phase of structures such as SSH-like 2D photonic crystals can be described by the Zak phase [see Xie et al., Phys. Rev. Lett. 122, 233903 (2019) – ref 40 in the manuscript], which is basically the integral of the Berry connection on the Brillouin Zone. In some works, such as the one by Xie et al. that is cited in our work, instead of the Zak phase the 2D polarization P is used, but the two are simply related by Z=2πP. The values of the Zak phase form a Z2 index in C4-symmetric topological crystals such as the one described in our work and can only take the values 0 or π for each direction for trivial or non-trivial topologies, respectively [see Benalcazar et al., Phys. Rev. B, 99, 245151 (2019)]. Following the calculations performed by Xie et al. we can safely conclude that the Zak phase for the directions (x,y) is (0,0) for the "compressed" structure and (π, π) for the "expanded" structure, meaning that the structures are topologically trivial and nontrivial, respectively. Analogous considerations can be found in references [14] and [39-43] of the manuscript that describe similar topological systems.

In our work, we did not perform the Zak phase or polarization calculation because we felt that it was not necessary, since Xie et al. and other authors [14, 39-43] are all coming to the same conclusion about the topology that accurately maps into our geometry. Besides, it would have made our paper difficult to be followed and distracted the reader from the novel and distinct results of our investigations.

We nevertheless acknowledge that the manuscript needs a discussion dedicated to clarify this point. We thus added the following paragraph to the main text:

*The topological invariant in SSH-like 2D photonic crystals like those described in this work can be classified by the Zak phase [40, 56], which is basically the integral of the Berry connection on the Brillouin Zone. In some works [40] the bulk polarization P is discussed instead of the Zak phase $\varphi_z$, but the two are simply related by $\varphi_z = 2\pi P$. The values of the Zak phase form a $Z_2$ index in $C_4$-symmetric topological crystals such as the one described in our work and can only take the values 0 or π for each direction for trivial or non-trivial topologies, respectively. [41, 57] It is known from the literature [40, 41] that in structures akin to those described in this work, the Zak phase for the directions (x , y) is (0, 0) for the compressed structure and (π, π) for the expanded structure, meaning that the structures are topologically trivial and nontrivial, respectively.*

Finally, it should be noted that this comment from the reviewer allowed us to notice a typo in Figure 2, namely, the order of the electric field maps was inverted (the low energy eigenmode was the high energy, and viceversa) for both compressed and expanded structure. The figure has now been revised.

*2. The paper states that the clearest indicator of a topological transition is the appearance of spatially confined guided modes at the boundary between domains of differing band topology, citing references [5, 7, 56–58]. However, upon reviewing reference 5, I found no discussion of this nature. It would be constructive for the reader to provide a precise reference that for such discussion.*

We thank the reviewer for pointing out a phrase in our manuscript that is likely to convey an overly simplified message. First of all, we toned the sentence down, by replacing "*The clearest evidence of the presence of a topological transition is the emergence of spatially confined guided modes at the boundary between two domains with different band topology.*" with "*One of the fingerprints of a topological transition is the so-called bulk, edge correspondence, that is the emergence of spatially confined guided modes at the boundary between two domains with different band topology*". This emphasizes that the

emergence of guided modes is one of the direct ways to determine that a topological edge mode has been achieved. This can be further supported by the calculation of the different Zak phase values of the two domains, as discussed in our answer at point 1.

Also, we believe that there was a mis-citation from the reviewer while requesting this clarification. Indeed, in reference [5], figure 5 and the relative discussion ("topological edge state") clearly show the presence of spatially confined modes at the interface between topologically distinct phases. Reference [5] is a seminal work for topological photonics, having been cited more than 1300 times, in particular from the researcher community that studies 2D SSH-like geometries.

References [7, 58-60] in the original text (now [56-58] in the revised manuscript) were specifically chosen because they demonstrate the presence of localized modes at the border between topologically distinct domains. It should be noted that the phenomenon that gives rise to the localized guided mode at the interface between two domains is universal and pertains to all these work, despite the nature of the topological photonic crystal can vary. For instance, reference [5] shows a spin-hall-like topological crystal, while reference [7] shows a valley-hall topological crystal. Spatially confined guided modes are therefore genuine characteristics of topological photonic crystals based on the 2D SSH-like geometry that we utilize in our work. As a reference, these topological edge states are demonstrated also in figure 2 in [39], figures 2 and 4 in [40], figure 3 in [41], and figures 1 to 3 in [42].

To further clarify this point to the general readership of the journal and to extend its implications, we added reference [1] (Hasan and Kane, Rev. Mod. Phys., 82(4), 3045 (2010)) to the bibliography. In this work, the origin of the localized mode is discussed both formally and with examples for electronic systems. The similarities between "electronic" topological insulators and topological photonic crystals are well known and discussed in many works, such as [2,3].

*3. Unlike reference 37, this study lacks experimental data comparison, which necessitates at least an examination of the numerical methods' fidelity. Details such as error estimation, mesh sensitivity, and convergence rate, along with parameters selected for the FEM simulations, would aid in reproducibility and validation of the numerical findings.*

We thank the reviewer for pointing out some missing details in the methods. We therefore added a dedicated section in the Appendices with the information that ensures reproducibility and validation of our calculations.

We chose to perform the simulations with a small mesh size to obtain precise results, but we avoided using a needlessly small mesh size that only lengthens the computational time with no advantages on the solution precision. We show in Appendix B that the results are practically unchanged despite shrinking or expanding the mesh elements with respect to the one that we originally used.

Notably, the convergence rate shows convergence to an error value generally smaller than $10^{-15}$ in just one iteration – except for the eigenfrequencies calculated at k=0, where the eigenfrequency is ~0 and the simulation reaches a convergence value of $10^{-7}$ in three iterations. This shows that the calculation is efficient and most importantly extremely precise.

The experimental validation of the results obtained in this work will also be the definitive confirmation that our simulations are valid. However, the realization of the topological device is beyond the scope of the present work that serves as a proof-of-concept. The fabrication of the resonator device is in program, but not yet started.

*4. Given the scalability of Photonic Crystals, any frequency range could be achieved with a change of scale. It is unclear why the THz range is specifically singled out. Including it in the title seems somewhat contrived to me.*

The motivation for the THz is fundamental and twofold. On one hand, as we described in the introduction of the work, the THz is a spectral range of extreme technological interest for many diverse applications, and according to our knowledge, there is no demonstration in the literature of potential topological systems based on group-IV semiconductors that work in the THz . This further strengthens the significance of our findings. In our view, mentioning the frequency range and the material platform directly in the title is very appropriate, not only for its technological but also for its foundational implications.

We would like to emphasize that there is an intrinsic limit in the working range of a photonic crystal, which is set by the optical bandgap of the materials composing it. In the case of Ge, at room temperature this value is around 0.66 eV, and it increases to 0.76 eV at cryogenic temperatures (the direct gap lays about 150 meV higher in energy). Light with energy above the bandgap is therefore absorbed by the photonic crystal itself, i.e., lost. The situation is even worse in the case of doped materials with intra-gap levels that can generate unwanted photon absorption. A Ge-based photonic crystal is therefore limited at best to 180 THz.

Furthermore, fabrication constraints also need to be considered. While it is true that the scaling invariance of photonic crystals could in principle allow any desired frequency, experimentally this is not necessarily true. The size of the microcrystal that is described in this work is about 600 nm, that is at the limit of the fabrication possibilities of optical lithography. Ge features having micrometer size and lattice parameter (2 μm in our work) have been already experimentally demonstrated in Ge-on-Si heterostructures, see e.g. ref [37, 38]. Dimensions can be further scaled down by relying on e-beam fabrication. However, there is only a certain amount of shrinkage that is experimentally feasible for shifting the operation towards higher frequencies: at that point the best solution would most likely be to work with a material with a lower refractive index.

5. References to "supplementary materials" on page 3 were noted, yet I couldn't find any of these supplementary materials in the submission.

We thank the reviewer for pointing out the problem with the supplementary material. We sincerely apologize. This was a mistake: somehow the supplementary material was not included in the merged pdf file upon uploading the material. The supplementary material has now been included at the end of the main text in the shape of appendices, to avoid any possible issue.

6. It would be interesting to mention the role of the edge (111) of the Germanium stand and what would be the behavior of the crystal in its absence.

The Ge crystal was modeled with both (100) and (111) facets because such a structure is known from previous works on the topic. The (111) facets are sub-wavelength and therefore offer a negligible role in the overall photonic properties. The shape of the high-refractive index element does not really influence the bandstructure nor the electromagnetic field distribution, as is shown in the literature for elements of different shapes and forms, such as squares [14], circles [39,40] or triangles [43].

Nevertheless, we performed a simulation of the same structure described in the work, but without (111) facets, i.e., a perfect Ge square, and we inserted the results in the supplementary material. It can be noticed that the bandstructure is identical, as is the electromagnetic field distribution. We amended the text by adding *"The {111} facets are sub-wavelength and their role on determining the photonic bandstructure of the crystal is negligible, as shown in Figure 5 in Appendix A."*

7. Minor typographical errors identified include "perfomed" which should be corrected to "performed," and "simmetry" to "symmetry."

We thank the reviewer for pointing out those typos, which we have thoroughly corrected.