Dear Editor,

Thank you for sending us the Referee report on our manuscript "Topological photonic band gaps in honeycomb atomic arrays". We thank the Referee for careful reading of the manuscript and for valuable comments and suggestions. We revised the manuscript accordingly and resubmit it with changes highlighted in blue. Below we present point-by-point replies to Referee comments and a summary of the changes to the manuscript.

Referee writes:

"1. When computing the Chern number in section 2.4 I would like the authors to discuss how the singularities around Gamma affect the Chern number calculation. Do they pose any problems in convergence?"

Our reply:

We comment on this issue on p. 9 of the revised manuscript. The role of singularities arising at $|\mathbf{k}| = k_0$ can be made clear by integrating Berry curvature over hexagonal areas of increasing side k_1 up to the maximum size corresponding to the Brillouin zone, and examining the behavior of the result around $k_1 = k_0$. The result of such an integration is presented in Figure 2 below. We also show Berry curvature as a function of $\mathbf{k}$ in Figure 1:

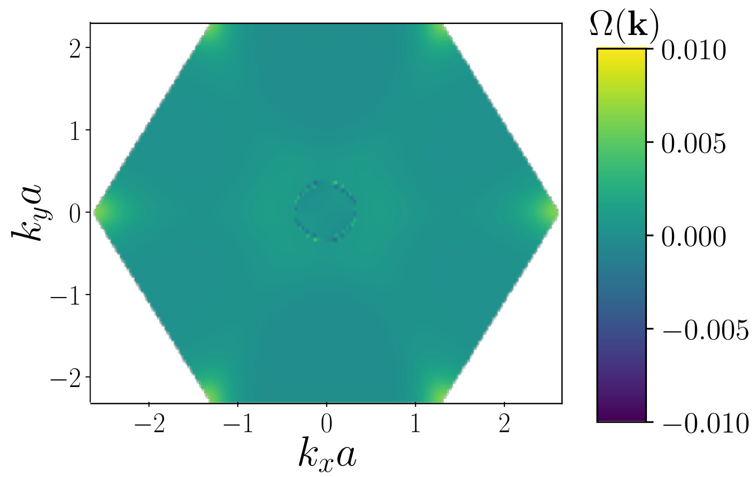

Figure 1. Sum of the Berry curvatures over the two lowest frequency bands.

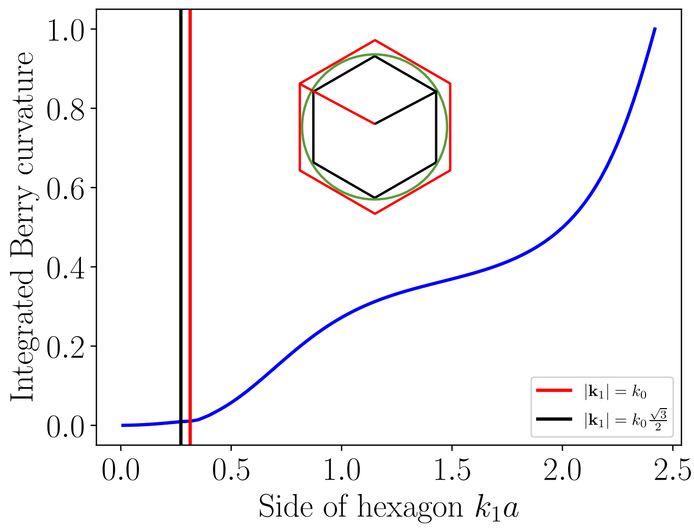

Figure 2. Integrated Berry curvature over a hexagon of side k_1. The two vertical lines represent the hexagon inscribed within the circle $|\mathbf{k}| = k_0$ and the smallest hexagon that contains the circle.

We observe that Berry curvature becomes different from zero for $|\mathbf{k}|$ around k_0, however without causing any particular problem in the integration over $\mathbf{k}$. The result of the latter grows monotonically with k_1 and converges to $C = 1$ when the integration area covers the full Brillouin zone.

Referee writes:

"2. In the same section, I am a little surprised that they don't make connection to the extensive literature of non-Hermitian topology. In particular the system in free space has a clear non-hermitian component due to the decay rate. In non-Hermitian systems one can also define Chern numbers. More generally, when the spectrum is complex, as it is the case up to section 3, one can define invariants in the space of eigenvalues ($\mathrm{Re}(E)$ and $\mathrm{Im}(E)$). Here however, the authors choose to define the Chern number as if the decay rate is not zero. I think the paper can improve substantially if the connection with non-hermitian topology is made."

Our reply:

We now make the link with the recent literature on non-Hermitian topology on p. 8, after Eq. (22). Indeed, topological properties of non-Hermitian systems may be richer than those of Hermitian ones and Chern number (21) is not always appropriate nor sufficient to analyze them. However, in our system the non-Hermitian aspect is rather trivial, concerns only a small fraction of states (those within the light cone), and the gap between eigenvalues on the complex plane can be identified as a real line gap. Thus, according to the previous studies [21,22], Chern number (22) is the appropriate topological invariant to consider.

Referee writes:

"3. Lastly, the authors say at the end that when $d < \pi / k_0$ it is hard to define topology. They also observe that the system becomes gapless. Hence, their remark that this complicates the calculation of insulators seems to be not a well posed question, since to define invariants for insulators one needs a gap. Perhaps they can clarify further what they mean. »

Our reply:

We assume that the Referee means "$d > \pi / k_0$" because this is where complications arise. We admit that our initial explanations for the reasons of these complications might have been unclear. We have now modified and appended them (see the new text on pp. 14,15 and 16). In short, propagating modes with k_z different from zero arise in the system with large d. Thus, 2D analysis in which the band diagram is studied as a function of k_x, k_y and Chern number is calculated by integrating over k_x, k_y, becomes insufficient. It remains to be seen whether a gap is still present in the system with large d and whether this gap is topologically nontrivial, but such an analysis is beyond the scope of the present work.

We hope that the above reply and changes to the manuscript are sufficient to make the latter acceptable for publication.

Sincerely yours,

P. Wulles and S. Skipetrov