Requested changes

1. *Maybe most importantly, the interpretation of the results should be sharpened. More precisely, what do the zero modes mean physically? To elaborate: in a hermitian setting, Majorana zero modes (MZMs) for example encode a ground state degeneracy due to a spatially split complex fermionic state, which in turn can be used for potentially robust qubits. Indeed, the authors state that the MZMs acquire a finite lifetime set by the dissipation \gamma. While the authors do not explicitly comment on it, this probably implies all information stored in dissipative MZMs to be effectively washed out within the time scale set by \gamma.*

Answer: We thank the referee for raising this valuable point. The zero modes we obtain in our setup are similar to those in a closed Hermitian system. This is evident both from the behavior we find, including the topological classification, but has also been established earlier, see, e.g., SciPost Phys. **6**, 026 (2019) and Phys. Rev. Lett. **124**, 040401 (2020). However, the zero modes we obtain also carry a finite imaginary part. Thus, in spectroscopic measurements (such as by scanning tunneling microscope), the zero modes will exhibit a broadened peak inside the bulk gap. Also, owing to the finite imaginary part of the MZMs, they will ultimately likely not survive in the long time limit. Thus, we agree with the referee that the information stored by MZMs might be lost on the long time scale, set by \gamma. Additionally, we believe our results might open up an avenue to further seek for MZMs in dissipative systems with designed zero or infinitesimally small imaginary parts, which will then also survive in the long time scale. We have now modified our manuscript to clarify this point of our results.

Changes in the manuscript: We have added the sentences: "*This finite imaginary part can be understood as the lifetime of the MZMs in a dissipative system and as such they would appear as a broadened peak in, e.g. spectroscopy measurements.* " and "*However, the MZMs now also carry finite imaginary parts and thus can survive only up to the time scale set by the dissipation strength γ. This constraint applies to all types of ZMs we obtain in this work.*" in Section 3.1.

2. *However, there is probably more to the story that is worth mentioning. The MZMs relate to edge states of the damping matrix X (note that the spectrum of the Liouvillian is governed by the damping matrix). As the authors correctly point out, the Liouvillian governs the time-evolution of the density matrix. But what is the steady state that the system evolves to? Which physical observable can be connected to the edge Majoranas (or the „robust zero modes" [RZMs]), and on which time scales (In the steady state? In the asymptotic approach towards the steady state? In an initial time range?)? The authors for example write at the end of Sec. 3.1 that the MZMs survive - yes, but what is the physical quantity related to them that survives?*

Answer: We thank the referee for this question. We do not explicitly investigate the topological properties of the steady state density matrix, which may also be interesting. In our study, we instead focus on the topological properties of the spectrum of the Lindbladian. In that sense, our study of the topological properties of the dissipative system is a transient property. But despite the transient properties, MZMs and RZMs can, in principle, still be detected in spectroscopic measurements and thus have physical consequences. One motivation of our choice is the natural connection between non-Hermitian physics and the Lindbladian, where the former has lately gained an immense amount of interest. We also note that, as discussed in PRL **124**, 040401 (2020), the topological properties of the spectrum and the steady state are unconnected. Still, it might be worthwhile also to investigate the topological properties of the steady state for our system. However, we leave that for the future, as in the present study, we are mainly interested in investigating the fate and emergence of the zero modes from the spectrum. We have now discussed these issues in our revised manuscript.

Regarding the survival of the MZMs, we want to add that one can still obtain MZMs in a dissipative system. It appears that what we originally stated was not quite complete, and we thank the referee for pointing this out. The MZMs in a dissipative system carry a finite imaginary part, which sets the lifetime of these states. The amplitude of the imaginary part increases with the increase of the dissipation strength \gamma. Thus, \gamma constrains how long the MZMs can survive. We have now modified this statement in our revised manuscript.

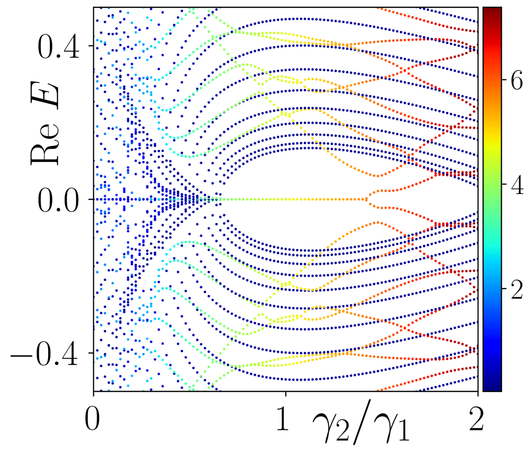
Changes in the manuscript: We have modified and added a few sentences in Section 2.2.1: **"***Further, we note that one of the differences between X and the NH Hamiltonians so far usually considered in the literature [55, 56] is that the imaginary part of the eigenvalue of the matrix X , i.e., the Lindblad spectrum Im E, is always greater than zero, which ensures that the density matrix decays over time and approaches its steady state ρ_{SS}: ρ_{SS} = exp [i L_+ t] ρ(0) as t goes to infinity [104]. While studying the topological properties of the steady state is also important [45, 47], in our work we focus on the spectral topological properties of the Liouvillian, as these are independent of each other [73] and the Louvillian offers the natural connection to NH physics. In the latter context we note that Im E ≥ 0 provides a 10-fold classification of the Lindbladians [73], in contrast to all NH Hamiltonians, which exhibit a 38-fold classification instead [66].*"

And also added a sentence in Section 4: "*In our study, we only investigate the spectral topological properties of the Liouvillian and a next step can also be to study the topological properties of the steady state.*"

Less important, but still to be discussed:

1. *The loss specified in Eq. (4) is definitely not a generic loss: it can be seen as loss of electrons with eigenvalue +1 of spin-sigma_x. This is the same spin component to which the magnetic field couples. Which of the results obtained are special to this type of loss, which are generic? It would be nice to have some comment on other effect of other spin polarisations of the loss (although there remains so much to be explore about truly generic couplings to environments that additional future studies seem part of the full answer).*

Answer: We thank the referee for this suggestion. We agree with the referee that the loss we consider is not fully generic and that it would be interesting to also consider different types of coupling with the environment. Based on the referees' suggestion, we have now checked the effect of a spin-polarized loss on our phase diagram. Notably, we do not obtain any dramatic changes to our results: We still obtain both the MZMs and the RZMs with only a slight modification in the phase diagrams. For completeness, we provide a illustrative plots of this behavior below. Now spin up and down sectors have different dissipations γ_1 and γ_2. We fix γ_1 to a specific value and show the real part of the Lindblad spectra as a function of γ_1/γ_2.

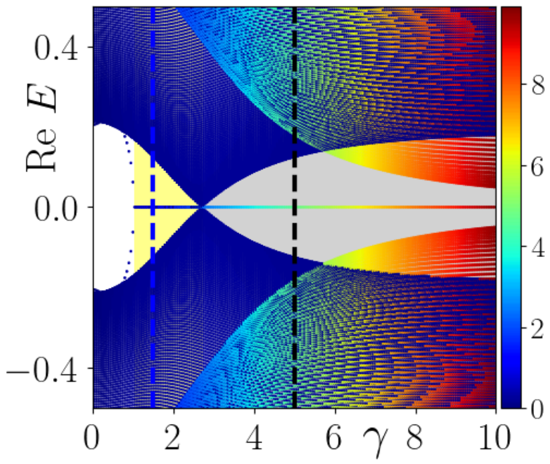


Changes in the manuscript: We have added: "*Furthermore, we also investigate a spin-polarized version of the loss operator (Eq. (4)), i.e., we consider different dissipation strengths for the two spin species. However, we do not observe any dramatic changes to our results as long as the spin polarization is not complete. In particular, we still obtain both the dissipation-induced MZMs and RZMs.*" in Section 3.2.

Also added: "*The form of loss may also be possible to adapt and it would be intriguing to find the form of loss by considering different types of coupling with the environment.*" in Section 4.

2. *In Fig. 3 (a), the MZMs seem show a splitting at large disorder. Are they split at all disorder strengths (maybe just very weakly so)? Is the splitting merely due to an increased overlap of the Majorana wavefunctions with disorder?*

Answer: As there is no disorder in Fig. 3, we assume the referee meant to say dissipation for Fig. 3 (a). The splitting at large dissipation strength is indeed due to an increased overlap of the Majorana wavefunctions. By considering a larger system size, we can eliminate this splitting. For completeness, we provide an illustrative plot showing no splitting for a longer wire.



Changes in the manuscript: We have added: "*We finally note that, in Fig. 3(a,c), the splitting of the MZMs for higher values of dissipation strength γ occurs due to finite overlap of the wavefunctions of the MZMs. As such this splitting disappears for longer NWs.*" in Section 3.2.

3. *In Sec. 3.2.1, disorder averages are performed. The authors should comment a bit more on what exactly they do. Should I think of the data shown in Fig. 5 as running different microscopic disorder configurations, then ordering the states in some form (e.g. the real part of their energy or so), and then averaging the eigenvalues with same ordering number over the disorder configurations? Also, it would have been easier for me to have the details of disorder averaging (e.g. 50 runs) in the main text, not the figure caption.*

Answer: The referee correctly points out how we perform the disorder averaging. We have now added a few more sentences in the text to explain this better for the reader. We now also provide the details of the disorder averaging in the main text.

Changes in the manuscript: We have added: "*We consider disorder averages over 50 and 500 random configurations of the disorder potential to obtain the disorder-average Lindblad spectra in Figs. 5 and 6, respectively, and also check for convergence in disorder sampling. To this end, we sort the eigenvalues according to* *their real parts in ascending order for each random disorder realization. If two or more eigenvalues have the same real parts, we also sort their corresponding imaginary parts in ascending order.*" in Section 3.2.1.

4. *Maybe related to point 3, why do the averaged RZMs in Fig. 6(a) all have identical imaginary parts? If one looks at individual disorder runs, do these states still come in pairs with identical imaginary parts within the pair?*

Answer: We thank the referee for noticing this. In the presence of disorder, the RZMs do not come in pairs with identical imaginary parts. However, while considering the average over disorder, we did not sort their imaginary part, and thus, the RZMs appear to have identical imaginary parts. We initially focused more on the real parts of the RZMs to show that they still survive in the presence of disorder. However, following the referees' comments, we now also sort the RZMs according to their imaginary parts and we have thus modified this figure in our revised manuscript.

Changes in the manuscript: We have modified Fig. 6.

5. *Grouping RZMs into pairs with identical imaginary parts, do both pairs have weight at both ends, or is one pair located at one end, and the other pair at the other end (if that were the case, why the asymmetry)?*

Answer: We have checked the localization properties of the wavefunctions of the individual RZMs (although, we consider a longer wire so that the RZM wavefunctions are spatially separated from each other rather than a linear combination of them). We observe that the RZMs with degenerate imaginary parts are localized at two opposite ends. We have now commented on this in the manuscript.

Changes in the manuscript: We have added: "*Furthermore, we observe that the RZMs with degenerate imaginary parts are localized at two opposite ends.*" in Section 3.2.

6. *Can the authors confirm that the disorder averaging converges after 50 configurations?*

Answer: We have checked the results for more disorder configurations (100,500,1000). We do not observe any dramatic changes apart from changes in numerical numbers after some decimal positions. Thus, we believe our result converges after 50 configurations. Nevertheless, for Fig. 6, we now provide results with a disorder averaging over 500 samples, while for Fig. 5, we consider a longer chain (500), such that there will also be self-averaging over the length of the chain.

Changes in the manuscript: We have considered a disorder average over 500 configurations in Fig. 6.

7. *Do the authors have any idea as to why the peaks of the end states are sometimes not right-left-symmetric? Is that a feature that has converged w.r.t. disorder configurations? Is only the maximal peak height different, but the integrated weight per side remains the same (which would be an edge-dependent smearing out)? Is there a shift of weight from one side to the other?*

Answer: We thank the referee for this question. We have checked the convergence of the result with the disorder configurations (see Question 6 above). We notice that we still get this left-right asymmetry even after considering many more disorder realizations. In fact, we obtain this left-right asymmetry even without disorder, as seen in Fig. 4 (b), and as such it cannot be a disorder effect. However, we do not know the specific reason for this asymmetry.

8. *In Fig. 2 (b), 4(c), 6(c), the dots for the MZMs are all red. Red is the end of the shown color scale for the imaginary part of the eigenvalue. Are the MZMs modes with maximal imaginary part, or are they just modes of „high“ imaginary part?*

Answer: The MZMs have reasonably high imaginary parts. However, they are not the states with the maximum imaginary part. We thank the referee for pointing this out. We have now added a sentence in our revised manuscript to point out this.

Changes in the manuscript: We have added: “*It is here also worth mentioning that, while the MZMs carry a reasonably high value of imaginary parts, they are not the states with the maximum imaginary parts.*” in Section 3.1.

9. *Can the authors say more about the robust zero modes (RZMs)? Could they for example identify their wave functions or energies analytically? Is there an analytical way to connect them to the exceptional points (the numerics are certainly quite convincing, but maybe that could help identify the reason for their robustness)?*

Answer: We thank the referee for this comment. We agree with the referee that it would be interesting to find an analytical expression. However, we do not currently know how to proceed analytically for this system, as it is substantially more complicated than e.g. the Kitaev model, and therefore we provide the numerical results only. Nevertheless, we now point out this as an interesting future avenue when discussing the RZMs.

Changes in the manuscript: We have modified a sentence in Section 4: “*In particular, if it is possible to gain more control over the emergence of these modes a priori, with the knowledge of the isolated system's Hamiltonian and the specific form of the dissipation and also connect the emergence of RZMs to EPs via some analytical expressions.*”.

Finally, I noticed a couple of minor issues - nothing dramatic, but let me just point them out.

1. *The authors use the formulation that the density matrix decays - that is a bit ambiguous. The density matrix preserves its trace. It is correct, however, that it evolves towards its steady state expression with an exponential time-dependence.*

Answer: We thank the referee for pointing out this ambiguity. We have now clarified this in our revised manuscript.

2. *In Eq. (1), the „+h.c.“ leads to a doubling of the hermitian terms (chemical potential, magnetic field). One could introduce an extra factor 1/2, or add the +h.c. only for the terms that need it.*

Answer: We thank the referee for pointing out this typo. We have now corrected this in our revised manuscript.

Changes in the manuscript: We have modified Eq. (1) in the revised manuscript.

3. *To be overly picky, the formulation „we consider uniform loss i.e., L_i \neq o \forall i“ below Eq. (4) would be more on point if the loss amplitude in Eq. (4) would be \gamma_i, and then one could set \gamma_i=\gamma in the main text and \gamma_i spatially-dependent as chosen in the Appendix*

Answer: We have now incorporated this suggestion by the referee.
Changes in the manuscript: We have modified Eq. (4) in the revised manuscript.