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I. RESPONSE TO REFEREE 1

We thank the Referees for taking the time to read our manuscript carefully and providing us with detailed reports. Below, we address the concerns raised and describe the changes introduced in the manuscript (all modifications, except for the additions in the bibliography, are marked in blue text). For the sake of convenience, we address the remarks of the Referee in quoting the full texts of the reports. For the same reason, we took a liberty to divide the report in parts, and answer them part by part.

- *This paper studies how this principle can be used to generate a cat state which is in an equal superposition of two states with highly distinct photon numbers.*

  - We now make clear in the manuscript that we denote cat state a superposition of two states distinguishable through measures of the modes' energy, but not necessarily an equal weight superposition. In this sense, we show that cat states are generic and not accidental, and that the topological dynamics separates the two components on arbitrary distances in phase space and at a topologically quantized rate. Our second main result concerns the relation between topology and entanglement. The topological nature of the coupling induces a high entanglement between the qubit and the driving modes. We have clarified these two points through a major rewriting of the manuscript.

- *The idea put forward in the paper, i.e., of using topological frequency conversion to generate cat states, has been proposed and studied previously in the literature, namely in Refs. [22] (and also in [24]). It is therefore not clear what constitutes the new discovery in the present manuscript. The present paper does provide a thorough analytic/numerical characterization of the cat states; however, I currently do not see a clear motivation for this analysis that would justify publication based on this characterization alone.*

  - We provided clarifications regarding the originality of our work in our author reply on 2023-04-04. On the technical side, our work is the first one to treat on equal footing the driving modes and the qubit by using a full quantum description of both modes, contrary to Refs. [23,25] (former Refs. [22,24]), which rely on Floquet theory and therefore treat one drive as a classical parameter of the Hamiltonian. Thus, on the technical side the methods employed are different. In particular, we develop an adiabatic approximation valid to all orders in the adiabatic perturbative parameter, which is original. This reveals the role of the dressed Berry curvature in the dynamics instead of the bare Berry curvature

    On the physical side, our full quantum treatment enables a quantitative description of (i) the entanglement between the qubit and the two quantum modes which was not provided before, (ii) the role of the quantum metric and of topological constraint on the nature of the adiabatic states, (iii) the time evolution of the quantum fluctuations of the modes' photon numbers and its relation with Bloch oscillations and Bloch breathing. None of these important properties of the dynamics of the model were discussed previously. In particular, we show that the entanglement between the qubit and the modes originates from the geometry of the coupling, which is characterized by the quantum metric. As a consequence, a topological coupling induces a high entanglement. This relation between topological coupling and high entanglement is a new result.

    Beside, let us stress that we denote as a cat state a superposition of two states distinguishable through measures of the modes' energy, but not necessarily an equal weight superposition. In this sense we show that any realistic separable non-cat initial state dynamically evolves into a cat state. In contrary Ref. [23] (former Ref. [22]) says that some well-prepared initial state evolves into a cat state, and Ref. [25] (former Ref. [24]) does not discuss the creation of a cat state by the dynamics but the time evolution of an initial cat state, which is a very different phenomenon. In short, we show that cat states are generic and not accidental. The understanding of the generic nature of the creation of cat states results from our precise description of the adiabatic projector, which is possible only within our fully quantum mechanical treatment of both cavity modes.

    Furthermore, we added several paragraphs in the manuscript to discuss the applications of our results in the context of Thouless pumping, in order to stress the physical relevance of the generic nature of cat states. This induces a breakdown of the topological quantization of the average pumped charge. In particular, our results show that the deviation from quantization contains a quadratic contribution in the quantum fluctuation of the driving phase which adds up to the known quadratic contribution in the drive frequency (see Ref. [36]). We added a paragraph in the conclusion of the manuscript summarizing how our results translate for a quantum drive of a Thouless pump.

- ► *I unfortunately do not see how the manuscript meets any of SciPost's 4 expectations (listed under acceptance criteria). I would reconsider the manuscript if (1) the authors clearly explain what constitutes their new discovery, in particular how it contrasts to, e.g., Refs. 22 and 24., and (2) how this discovery meets the acceptance criteria of Scipost.*

  - ▷ We believe that our discoveries meet the acceptance criteria 3 of SciPost Physics: "Open a new pathway in an existing or a new research direction, with clear potential for multipronged follow-up work". Indeed, the starting point of our work consists in opening a new pathway in the study of topological pumping by considering quantized modes coupled to a qubit instead of a classically driven qubit. In doing so, we define a new regime of topological coupling between quantum degrees of freedom, two bosonic modes and a qubit. Our change of perspective enables us to demonstrate that due to the topological nature of the coupling, a generic separable initial state splits into an adiabatic cat state at a quantized rate.

    In particular, in the context of Thouless pumps, this opens a new pathway in the understanding of the breakdown of quantization of the pumped charge due to the splitting in a cat state. One can build for instance on our results to propose protocols of preparation of the adiabatic states (which leads to a quantized pumping). We now present these perspectives in the conclusion.

    Moreover, we unveil topological constraints on the entanglement between the qubit and the modes. This opens a new pathway on the relation between topology and entanglement.

- ► *As I understand, the operators n1 and n2 (which distinguish the two halves of the cat state) do not directly correspond to physical observables, but are mathematical constructs introduced to conveniently describe multi-tone driven quantum systems. The authors do mention that they for instance emerge when treating the drives as quantized modes and working in the large-n limit. However, I think the manuscript would benefit greatly if the authors made the physical meaning of these observables more clear, and, even better, proposed ways to detect the cattiness.*

  - ▷ We thank the referee for this valuable suggestion. The operators n1 and n2 do correspond to physical observables, they are the number of quanta of bosonic modes, accessible through energy measures. We discussed how to identify them in a previous work (Ref [24], former Ref [23]), in which we proposed an experiment where they are proportional to the energy of microwave modes coupled to a superconducting circuit, and detailed a protocol to measure them. In order to clarify the physical meaning of the operators n1 and n2, we added the section 2.1.1. We now define a regime of topological coupling between two harmonic oscillators and a two level system. The operators n1 and n2 are the number of quanta of the harmonic oscillators. The "cattiness" is then accessible by a measure of the energy of the bosonic modes. We then explain in Sec. 2.1.2 under which conditions our model of quantum rotors accurately describe the dynamics of the bosonic modes.