We thank the Reviewer for having spent their time to revise our last version of the manuscript.

Here are the Reviewer's remarks followed by our reply highlighted in red.

I thank the authors for their detailed responses to my comments. However, I am left with the impression that, conceptually, the model and results closely resemble those found in the reference [Xie et al., Phys. Rev. Lett], which shares similar symmetries, topologies, and results including the same type of phase transition. The theoretical added value of the modifications remains unclear to me. While I acknowledge that using different materials and designs can indeed lead to new applications in industry, this suggests that the work may be more suitable for publication in specialized engineering journals.

Reply: We thank the Reviewer for having appreciated our efforts to improve the quality of the manuscript. While it is true that our findings benefit from recent advancements obtained in the field of topological photonics, we believe that our work is conceptually new and significant as it demonstrates a way to integrate into Si photonics topological concepts that tackle a technologically relevant spectral region.

Two additional points:

1. Despite the correction in Figure 1, my initial concern from the previous report persists. It is unclear how a trivial phase and a non-trivial one can be simply related by lattice translation. This criticism applies to reference 40 as well. Nevertheless, it would be beneficial to clarify this point.

Reply: We apologies if our explanations were not sufficient to explain to the Reviewer the role of lattice translation in defining the emergence of non-trivial phases. The whole point, nevertheless, revolves around well-accepted concepts successfully utilized by various authors in the literature. We have tried to further revise our manuscript to further clarify the physics by adding the following paragraph at line 137:

"*It is worth noting that the definition of the unit cells does not affect the eigenvalues of the system, i.e., the photonic bandstructure: the* compressed *and* expanded *cells generate the same bulk (infinite) structure. In other words, when the PC has no boundaries, the* compressed *and* expanded *cells can be mapped one into the other, and the choice of the repeating unit does not yield differences in light propagation. Yet the eingenfunctions, i.e., the electromagnetic field distribution associated to a given eigenvalue, can differ. More specifically, the degree of freedom that changes between the two structures is the parity of E_z, which is reflected by the change in the Zak phase from (0,0) in the* compressed *cell to (π,π) in the* expanded *cell. While this property does not manifest itself in the bulk, it has a physical consequence when an interface is realized between the two topologically distinct domains. Indeed, the presence of a junction that acts as a boundary implies a closing gap to connect the states having the same parity [58].*

This is one of the fingerprints of a topological transition and is generally referred to as bulk-edge correspondence, i.e., the emergence of spatially confined guided modes at the boundary between two domains with different band topology."

2. As clear from Appendix A, the role of the facet (111) is negligible. It's not clear what is the advantage of including it in the design. One can imagine a large number of small modifications that will turn out to be negligible. I believe that the simplest functional forms (like a perfect square) are preferred at least from a fabrication perspective. Of course, one can ask the question regarding the order of magnitude of perturbations such that the results are not impacted. This question might have a practical relevance in terms of how precise the fabrication process has to be. However, the purpose of the inclusion facet (111) doesn't seem to me to provide an answer to the question.

Reply: Indeed, we deliberately included the {111} facets in our discussion because of their practical relevance in terms of the fabrication process, being one of the surfaces that develops as a result of the self-assembled growth of bottom-up Ge crystals (see reference [35]). We nevertheless pushed forward our discussion pointing out that our findings are more general given that the {111} facets are not playing a significant role. This eventually demonstrates that our approach is more universal as it can be implemented also using other less-refined top-down fabrication processes.