I thank the referee for his/her careful reading of the manuscript. His/her remarks help me to improve the manuscript. Please find my answers in what follows. The manuscript changes for the future resubmission are marked in red.

1) p.4, 1st column, end of first paragraph:

The last sentence implies that the scenario of Ref. [8,9] is common in two-neutron halo nuclei. However, most two-neutron halos have shallow resonances (using the terminology of the current paper) in the neutron-core and neutron-neutron channels. They are bound by the Efimov effect, even though there might be only one state in the universal region (see the discussion in [4]). The scenario of Ref. [8,9] (no neutron-core resonance) may be applicable to ^{22}C but there is conflicting information from experiment.

This point is important and I thank the referee for this remark. In this paragraph I was referring to the vicinity of the end-point of the Efimov spectrum for $a < 0$ when $N = 3, D = 3$ where $s = 2$ for the external part of the wave function. This regime appears for instance in the Faddeev equations of Ref. [Nai23] (see sec. 6 of this last reference when $1/|a_{12}| \neq 0$). I do agree that the general reference to Halo physics may be confusing as indeed, it is not yet clear if there exists a borromean state with a two neutrons separation energy S_{2n} much smaller than the neutron-neutron virtual energy ($S_{2n} \ll |E_{nn}| = \hbar^2/(m_n a^2) = 119$ kev with $a = -18.6$ fm [Chen08]). As suggested by the Referee, ^{22}C may be a candidate if one follows the conclusions of Ref. [Acha13] ($S_{2n} < 110$ kev) but the analysis of section 2.6 of Ref. [Ham17] or the introduction of Ref. [Hon22] show that this upper bound can be even larger.

The standard scenario for two-neutrons Halo states is indeed the Efimov one. To avoid any confusion, I will suppress the sentences concerned

Nevertheless, the scenario where there is no neutron-core s wave resonance and which is exhibited in Refs. [Hon22, Nai23] is relevant in the context of non Efimovian but universal physics, described in this manuscript. In this case, ^{22}C is a good candidate if one considers the upper bound $|a_c| < 2.8$ fm for the neutron-core scattering length (see section 2.4 of Ref. [Ham17]). ^{6}He ($S_{2n} = 975$ kev) is another example with $a_c \sim 2.47$ fm as measured in Ref. [Haun20]. At unitarity, this corresponds to the regime of section IV-C.3 for a three-body bound state composed of a single resonant pair among the three particles. However, the law in Eq. (78) does not take into account the finite value of the neutron-neutron scattering length. The study of these particular halos and in particular the link between the three-body parameters and the nuclear interaction is beyond the purpose of this work. I will add a comment about these particular halos in section IV-C.3.

2) Sec. IIC:

The results on the occupation probability for the small hyperradius region are interesting. In [Hammer, Lee, Phys.Lett.B 681 (2009) 500-503] causality bounds on the occupation probability for large distances in the limit $E \to 0$ were derived. Are these bounds consistent with the results on the occupation probability for small hyperradii in the paper?

The bounds derived in this manuscript are the generalization of the one in Ref. [Ham09] for the two-body problem in arbitrary dimension and partial wave with the following equivalences:

Notation in the manuscript	Notation in Ref. [Ham09]
s	$d/2 + l - 1$
ξ_s	$a_{L,d}$
α_s	$-r_{L,d}/2$

but here, s is a continuous variable. Consequently, with $r \equiv R_0$:

- Equation (12) of Ref. [Ham09] is exactly the same as Eq. (47) of the manuscript.
- Equation (14) of Ref. [Ham09] is exactly the same as Eq. (28) of the manuscript. Note that for $s = 1$, I found $P_{<R_0} = 0$ in the limit of a bound state of vanishing energy.

Thank to this remark, I will add this reference and this mapping in the manuscript at the end of section III-C

3) While some references to similar results obtained using other methods are made in the conclusions no connections to actual physical systems or experimental results are made. Are the states discussed in the manuscript theoretical artifacts or can they be observed in experiment? A brief discussion of experimental prospects for the observation of the N-body resonances in the conclusions would be useful.

1. **The states discussed are not mathematical artifacts**

   The two key properties behind the formalism developed in this manuscript are, first the fact that a large class of near resonant states are almost separable in the hyperradius and the hyperangle in a region of intermediate hyperradius; and second that consequently, there is a kinetic barrier (when there is no Efimov effect) in the hyperradius coordinate in the separable region. These two features are already present for the generic case in Ref. [Fed94]) or Ref. [Mat91] and in the unitary limit for instance in Ref. [Wer08]. Even if this is not explicit, these properties are present in the other formalisms and this explains why they can lead to similar results.

   The contact condition which is here deduced from a log-derivative condition in the region of the kinetic barrier [when one uses the simple choice of Eq. (83)] follows from a very classical approach. From this point of view, there is no doubt that the spectrum laws have a physical meaning. Saying it differently, one can derive these laws without referring to a modified scalar product having in mind that there is a natural cut-off at short hyperradius, below which the wave function has no more a universal shape.

   Interestingly, non Efimovian three-body resonant states were already found in configuration space in the hyperspherical formalism with an explicit repulsive kinetic barrier in the hyperradius coordinate in Ref. [Saf13]. Nevertheless, the universal character of these states was not revealed (see the title of this last reference). In this point of view, the modified scalar product (which generalizes here the one introduced in Ref. [Pri23]) is of first importance as it is shown that the contact model leads to the same normalization than the one obtained with actual finite range interactions (i.e. the reference model) in the limit of vanishing energy. The example of Sec 7. illustrates this property. This is why the present formalism leads to the generalization of the bounds found in Ref. [Ham09] which have a general character. The normalization issue of non-square integrable contact states, solved here by using standard mathematical properties, is then not a technical detail nor a theoretical artifact.

   The contact model gives the external part of the shallow resonant states and not the interior part in the region behind the kinetic barrier where the actual interactions are sufficiently attractive to induce a bound or a quasi-bound state. This explains the universality of the results.

   I will add a paragraph in the introduction of section VII explaining the motivation in studying the square well model.

   As shown in this manuscript, the major consequence of the kinetic barrier is that two N-body parameters are always necessary in the modeling when a quasi-bound state is able to occur, i.e. for a positive detuning and $s > 1$ or even for $s < 1$, when the generalized effective range parameter is anomalously large. Otherwise universality is governed by one three-body parameter as shown in Refs. [Wer06, Wer08]. I will add a paragraph in the conclusion to insist on this general feature. However, similarly to the universal theory of the Efimov effect, the present formalism does not make the link between these three-body parameters and the actual interactions of the reference model.

2. **The states discussed can be observed**

   Example of non Efimovian resonant states (i.e. characterized by a real index s) are already observed but not at the unitary limit:

   - Three-body systems with one s-wave resonant pair of particles interacting with another one, giving $s = 1$ (^{6}He and ^{22}C Halo bound states, $D\overline{D}\pi$ quasi-bound state ...)
   - ^{3}He droplets near the threshold (brunnian N-body resonance). The variational computation of Ref. [Sol06] estimates that one has a brunnian shallow bound state for $N = 30$. In Ref. [Son22], an estimates was given for the width of the quasi-bound state (considered at $N = 28$) with the equivalence $\Delta = s + 5/2$. This equivalence is well understood in the context of the hyperspherical formalism:
     (a) in the generic case (absence of two-body s wave resonance) by using the results of Ref. [Fab79] with the following equivalences:

Notation in the manuscript	Notation in Ref. [Fab79]
N	A
	$N = A - 1$
s	$\nu = \mathcal{L} + 1/2$ (see Eq. 2.16)
	$\mathcal{L} = L + (3N - 3)/2$ (see Eq. 2.14)
Δ	$L + 3A/2$ (see Eq. 3.20)

     (b) At unitarity by using for instance Eq. (34) of Ref. [Wer06]

In the case of ^{3}He, the scattering length is rather small and negative ($-13a_0$ in atomic units in Ref. [Uang82]), so that the resonance occurs in the generic case. The present derivation gives an overall suppression factor $4^{-s}/\Gamma(s)^2$ in the ratio Γ/E not present in Ref. [Son22] where the reasoning was partly based on scaling properties. This enhances even more the life-time of the quasi-bound state for a large s. Interestingly, the present derivation point out the crucial condition $E < E_0$ set by the minimal hyperradius R_0 such that the state is separable for $\rho > R_0$. If this condition is not met, the universal law on the width is no more valid. In Ref. [Son22], the energy E_0 is estimated at the value 40 K which is much larger than the energy of the resonance $E = 0.0194 \times N = 0.56$ K at $N = 29$ in Ref. [Sol06]. One can also use the mean radius of the droplet in Ref. [Pand86] for an estimate of the order of magnitude of R_0 with $R_0 \sim 7.8$ Å and find $E_0 \sim E/2$. This shows that for a precise evaluation of E_0 one needs more informations about the many-body wave function than what is published in Refs. [Pand86, Sol06] and a more refined study is required to known if the universal law is relevant in this case.

This discussion about ^{3}He droplet and these equivalences will be added at the end of section IV-A-2

In both previous examples of resonances, i.e. single s wave resonant pair in nuclear halo states and ^{3}He droplets, the scattering length cannot be varied so that experimental results cannot be decisive to validate the universality issue (for instance, considering a quasi-bound state the two parameters laws can be always adjusted by using the two data: resonance energy and width). In this respect, the possibility of varying the scattering length for fixed values of the three-body parameters using ultracold atoms is promising for studying the universality issue. Experiments can be designed following the prediction of Ref. [Nai22] of three-body resonance in presence of a two-body p wave resonance and by using the (^{171}Yb-^{171}Yb-Cs) system as suggested in Ref. [Pri23]. Hence the next step which is clearly beyond the goal of this manuscript, is the determination of the universal spectrum for given values of the three-body parameters as a function of the two-body scattering length.

This experimental proposal and the associated comments will be added in the conclusion.

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