Dear Editor,

Please find attached the revised manuscript titled "*Markovian to non-Markovian phase transition in the operator dynamics of a mobile impurity*" We are grateful to the reviewers for their time and insightful comments. We have carefully considered all the feedback and detailed our responses below. We hope with these changes, the article is now ready for publication.

Yours Sincerely,
Dr. Dominic Gribben - on behalf of the Authors

Reviewer report (black).
Response to the Reviewer's comments (blue).
Changes in the manuscript (red).

Reply to Report 1

I think the physics is clearly explained in the manuscript and I do not have substantial questions to ask the authors, although I have a main concern: the scenario and somewhat the ideas investigated by the authors are not new, but already present in literature that it has not been properly acknowledged.

For example, there are at least the following papers that use the interplay of the velocity of an impurity and the background maximum velocity to investigate various aspects of transport and information propagation (and I leave aside the huge literature on impurities in cold atoms, see eg. Schecter, Gangardt, Kamenev, New J. Phys. 18 065002 (2016) and refs citing it)

⋮

This is surely a non-exhaustive list. In particular, the authors should be aware of PRB101 where very similar questions to those investigated in the submitted manuscript have been studied: Fig. 1 of the mentioned ref is essentially Fig. 1 of the submitted manuscript, then in section IV of PRB101 a manifestation of the "Markovianity" discussed by the authors is described, and notice that both Hamiltonian systems and circuits are discussed in PRB101. The authors' work is original in the perspective and tools used to investigate this scenario, but due to the similarities with the mentioned work it would be nice if the authors could connect with it.

Thank you for your valuable feedback and for bringing important references to our attention. We have carefully revised our manuscript to better acknowledge the existing literature, including the specific papers you mentioned.

Our work, while related, offers distinct and significant contributions in two key areas: First, we study explicitly the information dynamics and operator dynamics of the impurity model. In doing so, we identified a previously unknown phase transition between Markovian and non-Markovain operator dynamics. Second, we show this transition is manifest in the information accessible via certain channels and provide decoders of this information whose fidelity also observes the phase transition.

By integrating these new dimensions, our research offers fresh insights and advances the current understanding in ways not covered by previous studies. We believe these contributions are significant and help to delineate our work from existing literature.

New paragraph on Page 3 describing and citing the references pointed out by Referee 1.

Overall, I think the problem studied by the authors is of some interest and the manuscript is accessible to non-experts, but in my opinion the manuscript does not contain the substantial novelty required by the high standards of Scipost Physics. Hence, I look at Scipost Core as a more proper journal to convey these findings and there I can endorse the publication of this manuscript, provided my concerns are taken into account.

We thank the Referee for their time and will follow their suggestion and resubmit to Scipost Core.

Reply to Report 2

The manuscript is overall well written, clear and accessible.

We thank the Referee for their compliments and for taking the time to consider our paper.

To be honest, I am more impressed by the breadth of the work than the depth of its main finding, which, as summarised above, is rather unsurprising. The technical setup of Clifford brickwork + OTOC is rather formal from a physical viewpoint, and not ground-breaking in terms of mathematical craft. More importantly, I find the study design suffers from two shortcomings:

- **The impurity-medium interaction is modelled by a swap with probability p, and hence the fluctuation generated is stochastic (averaging over realisations). This choice looks quite artificial and arbitrary, and is not motivated or justified. Even within the Clifford framework there are many other possible choices. Their potential effect on the results was not sufficiently addressed.**

We appreciate the referee's comment, and in light of it we reproduced the main result of the paper but with the swap gate replaced by a random unitary chosen from the two-qubit Clifford group. Similarly to swap-gate interaction, the random gate is applied between impurity qubit and the nearest medium qubit at a rate p. The same transition and universality are shown in Figure 1 below. The results are identical because the role of the interaction in the transition is simply to spread operators between impurity and medium. The transition is tuned by a compensation between the impurity velocity and the operator spreading in the medium. The particular choice of a swap gate was to mimic the absorption and emission of quanta by the impurity into the environment.

We have clarified this motivation on page 5 and added a footnote describing the numerical confirmation.

- **If I guessed the meaning of eq (1) correctly (see below), the impurity performs a biased random walk. In my opinion this is an unfortunate choice. Indeed, the diffusive critical scaling could be also attributed to the stochastic diffusive broadening of the random walk, as well as the quantum butterfly front broadening. Therefore, the definition of the model introduces artificial noise that obfuscates the critical physics it tries to describe.**

We appreciate the referee for pointing out this potential interpretation. To rule out noise effects from the motion of the impurity, we present Figure 2 which reproduces Figure 6 of the main text but with a deterministic drift. The deterministic drift is implemented by considering the impurity position to be real-valued number $x(t)$ and to increase it by a fixed v_d each timestep. When an interaction between the medium and impurity occurs, the impurity is swapped with the nearest environment site, i, such that $|i - x(t)| < |j - x(t)|$ for all j. We again find the same transition and critical scaling.

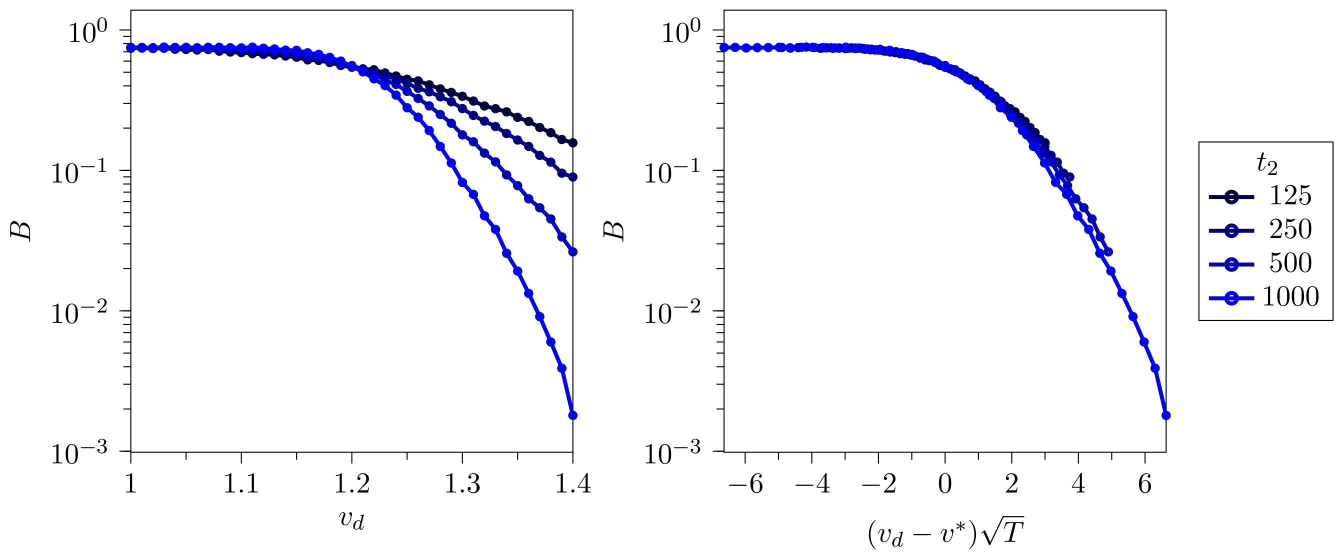

Figure 1: Left: Expanded view of the backflow phase transition near the critical point for a random unitary interaction. Right: Data collapse under a finite-time rescaling. Here we set $t_1 = 100$ and $p = 0.1$. This is the result of an average over 10^4 disorder realizations.

Finally, since the transition is interpreted as one between Markovian and non Markovian open dynamics, it would be more convincing to provide an effective description of the impurity dynamics, at least in the Markov phase.

We thank the referee for raising a potential confusion readers may have with the previous version of the manuscript. Importantly, we are not interpreting the transition as between Markovian and non-Markovian dynamics of the quantum state. We are discussing the non-Markovian dynamics of operators, as captured by an out-of-time-ordered correlation. This is in contrast to Markovian v.s. non-Markovian dynamics captured by time-ordered correlations, which is what readers might have in mind. To clarify this confusion, we added a new section making the comparison explicitly.

Here, we highlight the time-ordered correlations of the impurity are likely uninteresting. In particular, when the operator dynamics are Markovian, the time-ordered correlations will also be Markovian. In this limit, it should be sufficient to capture the dynamics by a depolarizing like channel that replaces the impurity with a random state conditioned on the initial state of the environment and its bare dynamics.

Added new section before conclusion highlighting distinction between operator Markovianity studied in this paper and notions of Markovianity in time ordered correlations.

In summary I find the manuscript publishable (after suitable revision, see below), but not significant enough for SciPost. SciPost Core may be a more suitable venue.

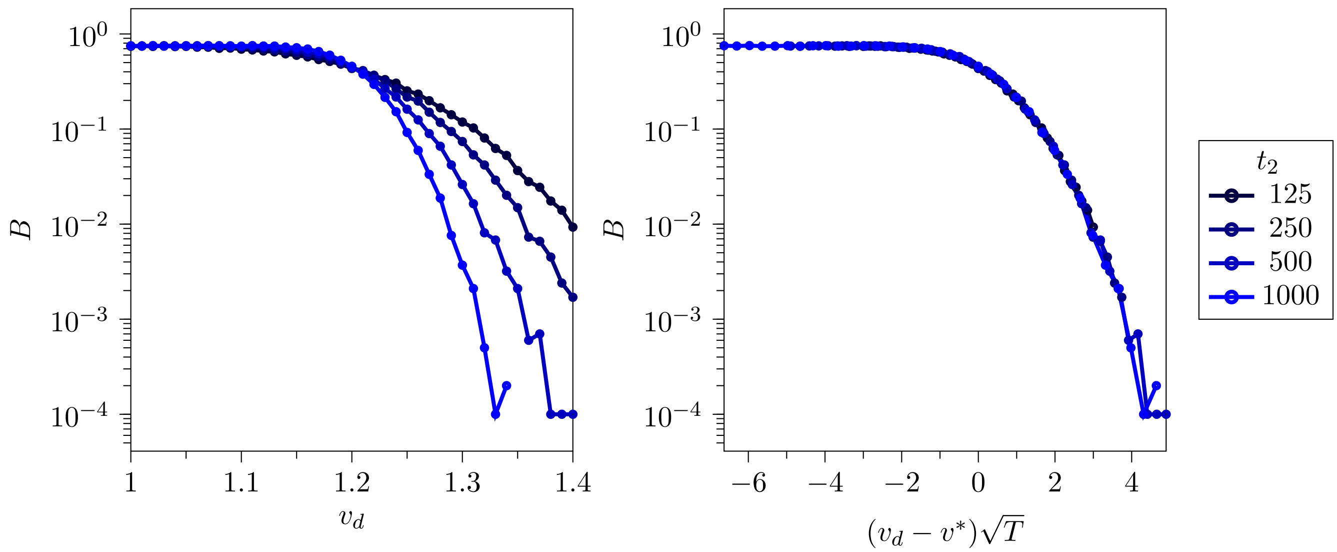

Figure 2: Left: Expanded view of the backflow phase transition near the critical point for a deterministic impurity drift. Right: Data collapse under a finite-time rescaling. Here we set $t_1 = 100$ and $p = 0.1$. This is the result of an average over 10^4 disorder realizations.

We thank the Referee for the recommendation, we will indeed be seeking publication in SciPost Core.

There are also a number of minor issues/questions:

- **In the "model" section, the definition of the models and the discussion on the physical motivation + anticipated results are too much intertwined. For the reader's convenience I find it preferable to restructure the section, and separate the definition from the discussion.**

We agree and thank the referee for bringing this to our attention, we have implemented their suggestion.

Restructured Section 2 as suggested with the models being fully introduced before discussing motivation + anticipated result.

- **Around eq (1) it should be clearly stated whether d is independently drawn for each time step (see above).**

Added a few words above Eq. (1) stating that d is indeed independently drawn for each timestep.

- **The paragraph below eq (1), where both v_B and c appeared, can be quite confusing (independently of the above point). If both veloicities have to be introduced here,**

it will be helpful to state that $c = 2$ is a strict upper bound for v_B, but $v_B = 1.2$ was known to be smaller than c. In fact I feel that it is better to postpone the introduction of v_B to the results section, where it truly belongs.

We thank the referee for this suggestion, we have made the change suggested and agree it does make this discussion clearer.

Removed mention of v_B from the Section 2, it is now introduced at the beginning of Section 4.1

- **In Section 3, it can be a bit confusing that the basic OTOCs (2) and (3) are defined but somehow discarded/amended (in section 3.1) by more a more complex protocol. The latter deserves to be better motivated than essentially quoting several references. The OTOC $B(v_B, t)$ could be more clearly defined (since that is the one that will be used).**
- **Related, in figure 4, which illustrates $B(v_B, t)$, it will be helpful to mark where there the involved operators act. Instead, information about Alice, Bob and Charlie is useful only in Section 5. So a figure showing their roles should rather accompany Figure 8.**

We thank the referee for raising these valid points. We have addressed them as follows.

Added explicit definition of $B(v_d, t)$, now Eq. (4), and updated Figure 4 to better highlight the significance of the protocol in measuring backflow. The original Figure 4 is now an inset to Figure 8.

- **In the conclusion, the authors stated that "Unlike the MIPT, but similar to the scrambling transition, this transition does not require the post selection of measurement outcomes." This phrase can be misleading. The standard MIPT is formulated without post selection, that is, upon averaging over outcomes. Observing MIPT requires obtaining the same outcome many times or overcoming the sampling problem in another way. But these two facts should be clearly distinguished.**

We agree with this statement and have amended the text accordingly.