Reply to the reviewer of the paper '*Precision magnetometry exploiting excited-state quantum phase transitions*'

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Dear editor and reviewer,

we understand the referee's issue, and therefore we have amended the claims according to the referee's suggestions. We also drawn new plots as suggested by the referee. The plot of F/N^2 requested in the report is the new figure 11. In the report attachment, the referee also asks for the plot of $\mathcal{F}_{h,m}/N^2$ instead of $\mathcal{F}_{h,m}$ in figure 4. We guess the referee meant $\mathcal{F}^*_{h,m}$ as this is the quantity plotted in figure 4. $\mathcal{F}^*_{h,m}$ in figure 4 (as well as $\mathcal{F}_{h,m}$ in figure 3) is plotted as a function of N in log-log scale: there are not curves $\mathcal{F}^*_{h,m}$ and $\mathcal{F}_{h,m}$ for different N. The plot for $\mathcal{F}^*_{h,m}/N^2$ similar to that in figure 4 is simply the same as $\mathcal{F}^*_{h,m}$ with slope $\xi - 2$ instead of ξ. We therefore plotted $\mathcal{F}^*_{h,m}/N^2$ as function of h and $\mathcal{F}_{h,m}/N^2$ as function of the eigenenergy for different N in figure 12. Notice that $\mathcal{F}_{h,m}$ is the maximum of $\mathcal{F}_h$ over all h (the peak value exemplified in figures 2(a) that corresponds to fixing $h = h_c^k$), then it depends only on the eigenenergy and on N. Similarly, $\mathcal{F}^*_{h,m}$ is the maximum of $\mathcal{F}_h$ over all eigenenergies (the peak value exemplified in figures 4(a,b) with $E_k = E_c$), then it depends only on h and on N.

Figures 11 and 12 show that $\mathcal{F}_h/N^2$ overlap for different N except around the peak, whose value slowly increases with N. This is compatible with the fit of the peak values $\mathcal{F}_{h,m}$ and $\mathcal{F}^*_{h,m}$ which show a power law with exponent slightly larger than 2. We do not enter the question whether these scalings are signature of the excited state quantum phase transition, as this is not the scope of our manuscript. Moreover, we do not think that this discussion is fundamental for presenting out results in the field of quantum metrology, as already acknowledged by the referee. For these reasons and in order to not weight down the main discussion, we commented on the new figures in appendix B and we amended the claims as suggested by the referee.

We think that we have addressed the criticisms of the referee, and that the new resubmitted manuscript is suitable for publication.

LIST OF CHANGES

- The sentence "$\mathcal{F}_h$ exhibits a sharp peak close to the critical energy E_c, and its maximum value [...] increases with the system size N" has been

replace by "$\mathcal{F}_h$ exhibits a sharp peak close to the critical energy E_c [...]", dropping the reference to the size scaling, and we have consistely reworded the beginning of the next sentence.

- The sentence "We suggest that the superextensive peaks of the QFI [...] are a signature of the ESQPT" has been removed, and we have consistely reworded the beginning of the paragraph.

- We have changed the title of appendix B, and discussed the new figures 11 and 12 there.

Yours sincerely,
Qian Wang and Ugo Marzolino