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Response to Report 1

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We thank the referee for taking time to review our work and to give a valuable report. Below we make a point-to-point response to the comments. We will make corresponding revisions according to these comments and suggestions, when the modification of our manuscript is opened by the Editor.

Comment: The paper deals with the problem of theoretically computing the ground state of a quantum dot coupled to a Majorana zero mode.

I do appreciate that the authors provide analytical solutions to this problem. There are however several major problems with this manuscript:

Reply: We thank the referee for appreciating our analytical solutions.

Point 1: There have been several publications earlier about coupling a Majorana bound state to a quantum dot, e.g. Refs. [22, 57] cited in the paper, or e.g. https://doi.org/10.1103/PhysRevB.96.201109 . The paper does not discuss at all how the results relate to any of the previous work.

Reply: We thank the referee for this valuable suggestion. The papers mentioned by the referee (Refs. [22, 57] and PRB **96**, 201109 (2017)) indeed studied the QD-MZM (quantum dot-Majorana zero mode) coupling system. However, they did not find the spin up to spin down transitions, which has remarkable influence on experimental detections (see Reply to Point 3).

Ref. [22] and PRB **96**, 201109 (2017) respectively used the QD-Majorana nanowire model and the effective QD-MZM model. The two works both found that the nonlocal behavior of topological superconductor can be measured through the lead-QD-MZM systems. The nonlocality relates to seperation between a pair of MZMs, which is influenced by the length of Majorana nanowire. In our manuscript, we consider the pair of MZMs are well seperated (the Majorana nanowire is long enough), thus only one MZM is coupled to the QD.

Ref. [57] studied the Kondo effect in normal lead-QD-MZM systems. It corresponds to the case that $T < T_K$, i.e. the temperature is lower than the Kondo temperature. The Kondo temperature is exponentially decreased by decreasing the normal lead-QD coupling. In our manuscript, the weak lead-QD coupling corresponds to a very low T_K and the condition $T >> T_K$ is easily met, thus our study is not relevant to the Kondo regime.

In the revised manuscript, we will cite the paper [PRB **96**, 201109 (2017)]. We will also add the discussions to Sec. 2 (Model and formula) of our manuscript.

Point 2: The model studied seems too simple for me: (i) in principle a Majorana bound state can couple to both spins in the quantum dot (see Ref. [22]), and (ii) the authors neglect the coupling to the superconducting continuum (at finite energy!). These simplifications are not discussed.

Reply: We thank the referee for the suggestions, and we here provide the discussions on these simplifications.

(i) In the model of our manuscript, under a $-z$-direction magnetic field $-V_Z\sigma_z$, we simply set the MZM just couples to spin z. In fact, Ref. [22] used a more realistic Majorana nanowire with also a z-direction magnetic field $B\sigma_z$, and made detailed analysis on the coupling spin. They found that "In the limit of large B the Majorana spin orientation at the edge becomes polarized along $-B\hat{z}$", which means that the MZM is almost just coupled to the $-z$ spin channel. We will adapt this to state the rationality of our simplification. For a large $-V_Z\sigma_z$ term (which is usually the requirement to generate MZMs), the MZM almost just couples to spin $+z$, as has been set in our manuscript.

Also, note that there are only two spin-dependent terms in the Hamiltonian: the Zeeman term and the MZM-QD coupling. If V_Z is small and the MZM couples to both $+z$ and $-z$ spin $ad_z+bd_{\bar{z}}$ (a, b are normalized coefficients), one can rotate the spin basis as $d_\uparrow = ad_z+bd_{\bar{z}}$. In this new basis, the MZM only couples to spin up, and the spin direction for Zeeman term is a bit deviated from z direction. In the meantime, the other terms of Hamiltonian is not influenced. If $V_Z = 0$ and the Zeeman term is absent, setting that MZM just couples to $d_\uparrow$ has no influence to any other term of the Hamiltonian.

To sum up, if V_Z is high or $V_Z = 0$, setting that MZM just couples to $d_\uparrow$ is very valid. Even if V_Z is a nonzero small value, the setting can also be used, with just the Zeeman term direction deviated a bit from z.

(ii) We focus on the MZM at zero energy, which is not affected by the superconducting continuum outside the gap. In fact, Ref. [22] and PRB **96**, 201109 (2017) have studied very similar problems, but using two different models. PRB **96**, 201109 (2017) effectively set that the QD couples to MZMs, but not coupled to the superconductor, which is similar to our manuscript. Ref. [22] set that the QD couples to a Majorana nanowire, which is a realistic model that consists coupling to both MZM and the superconductor. The results of these two studies are consistent, which also means that the coupling to superconductor can be neglected, as we have set in our manuscript.

In the revised manuscript, we will add these discussions to Sec. 2 (Model and formula) of our manuscript.

Point 3: I am also missing a discussion of experimental relevance of the findings. The authors talk about a phase transition, but in the end it's just about whether the ground state spin is up or down. What is the relevance of this?

Reply: The experimental relevance can be reflected by the DOS and MZM weight W, especifically Figs. 5(c, d) of our manuscript.

In Fig. 5(c), as the phase transition happens at $\epsilon_0 = \epsilon_c$ ($\epsilon_c \approx -U$), the DOS intersect at zero energy, and the MZM signal is sharply changed around the transition point. Although the MZM already exists and has a strong signal for $\epsilon_0 < \epsilon_c$, its signal is quite weak and hard to be detected when ϵ_0 becomes higher than ϵ_c. This corresponds to the sharp decrease of its weight as shown in Fig. 5(d). As ϵ_0 is further increased, the MZM weight becomes apparent again, as shown in Figs. 5(c, d). Our result is consistent with Fig. 3A of experimental result by Mourik et al. in Majorana nanowire Ref. [23]. By regulating the gate voltage, one section of the nanowire can be controlled as a QD. The gate voltage corresponds to $-\epsilon_0$ in our work, and the increasing of gate voltage in Fig. 3A of Ref. [23] corresponds to decreasing ϵ_0 in Fig. 5(c) in our work. As gate voltage in Fig. 3A of Ref. [23] is increased from -10 V to 0, the MZM signal is first weakened to be almost invisible. Then as the nonzero-energy states cross at zero energy (corresponds to our phase transition), the MZM signal becomes obvious. These experimental results are consistent with our results Figs. 5(c, d) with decreasing ϵ_0. They also indicate that even if the zero-bias peak is absent, we can not definitely judge that the MZM is absent.

On the other hand, in Lines 325-333 of our manuscript, we demonstrate that the DOS change along with the QD phase transition is very similar to the topological phase transition: They both exhibit the crossing of non-zero energy states, and the MZM signal seems to be absent before crossing but remarkable after crossing. This also means that the MZM does not necessarily induce a zero-bias peak.

In the revised manuscript, we will emphasize the experimental relevance of our results in Sec. 4 (Phase transition with Zeeman term) of our manuscript.

Point 4: The authors introduce a normal lead. However, in the approximation they use, the lead just gives a finite broadening to the states. To me, it seems just a complication to compute the Green's function - the eigenstates give exactly the same physical information.

Reply: We agree that the lead just gives a finite broadening to the states. However, this broadening is essential to demonstrate the change of MZM weight and the inspiration to experimental detections.

In Fig. R1(a), we plot the energy of eigenstates versus ϵ_0. This corresponds to Fig. 5(c) of manuscript and can be obtained no matter the normal lead coupling is present or not. For clarity, we also show Fig. 5(c) of manuscript in Fig. R1(b), which can be obtained only when the normal lead is coupled to QD. Indeed, the eigenenergy Fig. R1(a) is consistent with the spin-resolved DOS Fig. R1(b). However, compared to Fig. R1(b), Fig. R1(a) lacks the weight information: In Fig. R1(a), one finds that a zero-energy state always exists. Only in Fig. R1(b) when the QD couples to normal lead, one can indentify that the MZM weight changes violently versus ϵ_0, and notice that the phase transition plays an important role on the visibility of MZM signal.

Therefore, the coupling of normal lead provides the weight information, which can not be obtained by just solving the eigenenergy. On the other hand, the normal lead is usually demanded in MZM detections, thus introducing the lead is natural and consistent with experimental conditions.

In the revised manuscript, we will add these contents to the appendix.

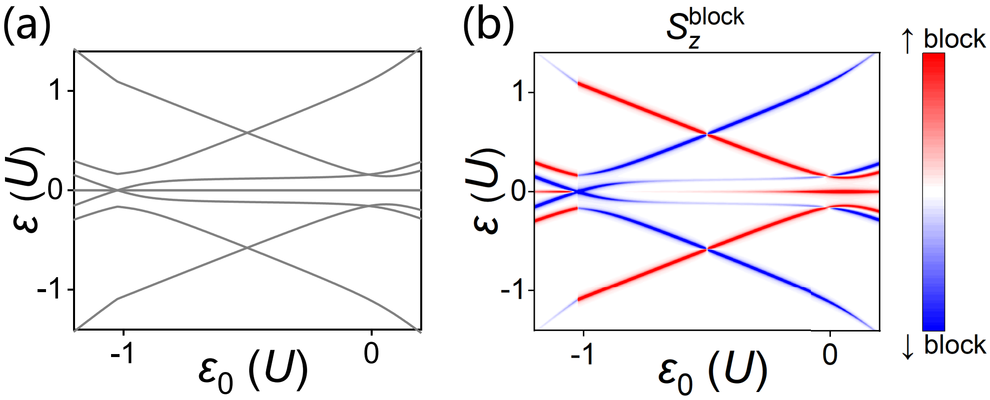

FIG. R1: (a) The energy of states versus ϵ_0. (b) In the presence of a normal lead, the DOS information versus ϵ_0 (the same data as Fig. 5(c) in the manuscript, but the colorbar is adjusted for clarity).