## **RESPONSE TO REFEREE #2**

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(1) "1. The authors preformed an optimization of the average power output of continuously driven engine. This is an important figure of merit, however, since the engine is composed of a quantum system fluctuation are expected to play an important role. If the fluctuations are large relative to the average, then the engine should not be really useful. I think it is important to showcase the fluctuations in power or work in order to evaluate the usefulness of optimizing the average power."

The fluctuations can be included in the formalism; I have added a new appendix that shows how this can be done. It can be seen how some extra computations need to be done, but they amount to solving equations that are analogous to the ones required for including the power or the dissipated heats. The computational cost is therefore larger, but it does not increase in complexity or scaling. I have not performed new calculations adding a term depending on the fluctuation; this will be done in a future work.

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(2) "In addition, from various works in the field of quantum thermodynamics related to thermodynamics uncertainty relations. It appears that there should be a tradeoff between optimized power, minimization of fluctuation of power and minimization of entropy production. I think the model studied allows to test and showcase this tradeoff, I think this will greatly benefit the field of quantum thermodynamics and contribute to the novelty of the article."

I have made it more clear in the text that the method described permits to include all those possible terms in the definition of the merit function, allowing for a combined optimization. However, I understand that the referee probably demands a calculation showcasing that possibility. In my opinion, that should be left for another publication that would focus more on the particular model and the physics of it, rather than on the methodology. In order to demonstrate the feasibility of the methodology, I think that the examples shown should suffice, and adding more results would unnecessarily lengthen the paper and obscure its message.

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Response to:

(3) "In Eq. (42) the Lindblad operators correspond to the Master equation without any driven, and are independent of the drive. This seems to be inconsistent with the statement that the Floquet-Lindblad Master equation is the appropriate way to treat the the open system dynamics of a periodically driven system. In the Floquet-Lindblad Master equation the Lindblad jump operators are transition operators between the Floquet states, which don't correspond to the transition operators in the static case."

and

(4) "In Ref. [32], there is a discussion of the validity regime of the use of the Floquet-Lindblad (sec. 3.3.1), can the author comment how these considerations relate to the present work."

Eq. (42) is the phenomenological model of a system, using a driven GKSL equation. The decoherence terms of GKSL equations are sometimes derived microscopically (by analizing the coupling to the bath and taking approximations), but often they are just given as phenomenological assumptions, whose associated rates can then be adjusted. This is the case of the model used here, and by other authors (for example in Refs. 15, 16, 17). In this kind of phenomenological approximation, sometimes the jump operators are considered to be independent of the drive.

In any case, the referee is right, and I should not have used the term "Floquet-Lindblad equation" to refer to Eq. (32). That equation, as it is presented, is any periodically driven time-dependent Lindblad equation. Since the name "Floquet" is often associated to the study of periodic phenomena in general, I carelessly used the name "Floquet-Lindblad" equation to refer to any periodically-driven GKSL equation. However, the term should be reserved to the form given for example by Eq. 33 in the reference that the referee points out ([32]). That equation does originate from the periodic Lindblad equation, but has been transformed by making use of Floquet's theorem, and has been taken to the Floquet reference frame. In fact, in the form given in Eq. 33 it is not explicitly time-dependent (the timedependency is implicit in the gauge transformation).

Therefore, I have removed the term "Floquet-Lindblad". And, regarding the validity of the Floquet-Lindblad equation, the considerations in Ref. [32] no longer applies. However, the general considerations about the GKSL (Markovian) approximation do apply: Born approximation (slow correlation between the system and the bath); Markov (fast decay of the excitations in the bath), and rotating wave approximation (neglect of fast oscillating terms with respect to the time scale of the system).

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[MINOR] 1. "I think referring to a continuous engine as an Otto engine might be misleading, as the Otto cycle is constructed by four well-defined strokes.

I have left the word "Otto" when referring to previous works that chose to use that terminology for two-strokes engines (the time of the two adiabatic segments is taken to be zero). But I have removed the word Otto from the rest of the places where it appeared, to avoid the confusion, as suggested by the referee.

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