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I. RESPONSE TO REFEREES

We thank the referees for taking the time to read our manuscript carefully and providing us with detailed reports. Below, we address the concerns raised and describe the changes introduced in the manuscript (all modifications, except for the additions in the bibliography, are marked in blue text). For the sake of convenience, we address the remarks of the Referee in quoting the full texts of the reports. For the same reason, we took a liberty to divide the report in parts, and answer them part by part.

A. Response to Referee 2

- *The authors state that their adiabatic expansion scheme is valid to all orders in the modes' frequencies. However, in the Appendix where the expansion is provided, it is noted that the expansion is asymptotic (i.e., that it will diverge beyond a certain order for any finite parameter). To some, the statement that the expansion is "valid to all orders" could imply that the expansion converges for any choice of input parameter. I therefore suggest that the authors clarify in the main text that the expansion is asymptotic.*

  - We thank the referee for the valuable comment. We now precise in section 3.1 that the adiabatic states are constructed as an asymptotic series.

- *The rotor model considered in Eq. (4) is equivalent to the classical drive model, since the large-photon-number limit is taken (i.e., the Hamiltonian in Eq. (4) is equivalent to the Sambe (or extended Hilbert) space representation of the multi-mode classical drive problem, e.g. considerd in Ref. 15. This seems to contradict the authors claim that their treatment is "fully quantum" in contrast to earlier works. I suggest the authors modify their wording to clarify that the model they consider is equivalent to a classical-drive model, and has also been studied in earlier works. The difference to earlier work is that they consider the quantum correlations of the classical drives [in the sense that a classical model is recovered from a quantum model by taking the $\hbar \to 0$ limit, even while the Schrodinger equation still describes the system in this limit].*

  - Our treatment is fully quantum mechanical. It is a well-known property of quantum Hamiltonians of the form $\omega\hat{n}$ that if the corresponding system is prepared in an initial state that is an eigenstate of the conjugate $\hat{\phi}$ operator with eigenvalue ϕ_0, it evolves in such a way that it remains such an eigenstate at any later time t, with eigenvalue $\phi_0 + \omega t$. Therefore, there is a close parallelism between the quantum evolution and the classical one, as far as the phase variable ϕ is concerned. We emphasize that this holds for any value of the Planck's constant, and not just in the classical $\hbar \to 0$ limit. Therefore, there is no inconsistency between our claim and the fact that we get the same global Hamiltonian as in the Sambé treatment of a periodically driven quantum system with a classical drive. We emphasize that our quantum mechanical description of the modes enables to describe the dynamics of any initial quantum state of the modes, and not only the eigenstates of the phase operator, which are the only states well described by the classical evolution. For these reasons, we feel that the referee's statement that the model is "equivalent to a classical-drive model" is potentially confusing. To clarify the issue at stake would have required to add several sentences to a paper which is already long. Therefore, we have not modified the text, following the line of our above answer.