Reply to the Report

October 30, 2024

We thank the Referee for his valuable suggestions and remarks. Below, we will address the issued raised by the Referee.

1. We believe that our procedure can be applied to extract the asymptotic behavior of a more general class of volumes beyond those considered in the present paper. For example, we could apply our procedure to certain deformations of JT gravity [1] or supergravity, where we would have a theory dual to a matrix model. In this context, the partition function in the bulk can be obtained by gluing a geometric volume to a "trumpet" that connects the volume to the AdS asymptotic boundaries. The functional form of the "trumpets" will, in turn, determine the form of the function $f(b_i, z)$ in the following formula for the asymptotics of the general volumes:

$$\mathcal{V}_{g,n}(b_1, \ldots, b_n) \propto \oint_0 \frac{\mathrm{d}z}{z} \frac{1}{(V_{\text{eff}}(z))^{2g-2+n}} \prod_{i=1}^{n} f(b_i, z), \tag{1}$$

where $V_{\text{eff}}(z)$ is the effective potential of the matrix model. In general, the only other caveat regarding the application of our procedure lies in the fact that every formula for instanton contributions presented in the paper holds only for a single-cut matrix model; thus, our analysis must be refined for more general spectral densities.

2. The non-perturbative effects considered in the paper are associated with eigenvalue tunneling in the matrix model, where all but one eigenvalue reside in its perturbative cut, while the remaining eigenvalue is placed at a non-perturbative extremum of the effective potential. The location of such an extremum, denoted as $\bar{z}$, is outside the principal cut and is naturally determined by the equation:

$$V'_{\text{eff}}(\bar{z}) = 0. \tag{2}$$

As noted by the referee, an instanton-like structure emerges from our formulas when we perform a saddle-point analysis of the z integrals by choosing the appropriate contour that includes the point $\bar{z}$. These effects can be interpreted from the perspective of minimal strings. It is well established that they are associated with ZZ branes. This reinterpretation was carried out in the context of JT gravity in [2, 3], but to our knowledge, a direct investigation in the supersymmetric case is still lacking. Nonetheless, we expect the presence of an analogous structure.

3. We believe that the answer is affirmative. The point is that the universality of the behaviour of the spectral form factor at late times originates in the matrix model from the universality of the form of the correlation function $\langle \rho(E)\rho(E') \rangle$ in the limit $E \to E'$ featuring the sine kernel [4]. In the matrix model this result is non-perturbative and it is very much related to our conjecture since in order to obtain the asymptotic formulas, in Appendix B we precisely analysed these kind of correlation functions (actually their

Laplace transform). In particular by repeating the procedure of the Appendix B we deduce that the i-th order subleading contribution to the asymptotic formula can be obtained by the analysis of i-th order term of the perturbative series attached to the first instanton and viceversa. Therefore we believe that it is very likely that the asymptotic formulas contain the information about the late time non perturbative behaviour of the Spectral Form Factor.

Request Changes:

Concerning the changes requested by the referee, we have added a paragraph before formula (2.25) (formerly (2.24) in the previous version of the paper). The purpose of this paragraph is to better justify the origin of this formula and its derivation. Moreover, before formula (B.1) and in footnote 7, we have attempted to provide a similar justification for formula (B.1), also drawing on the similarities with (2.24). Finally, we have briefly discussed the derivation of (B.7). Providing a self-contained derivation of this equation is quite challenging and would require repeating a substantial portion of the content from reference [2] in the appendix.

References

[1] Edward Witten. Matrix models and deformations of jt gravity. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 476(2244), December 2020.

[2] Bertrand Eynard, Elba Garcia-Failde, Paolo Gregori, Danilo Lewanski, and Ricardo Schiappa. Resurgent asymptotics of jackiw-teitelboim gravity and the nonperturbative topological recursion, 2023.

[3] Phil Saad, Stephen H. Shenker, and Douglas Stanford. Jt gravity as a matrix integral, 2019.

[4] Madan Lal Mehta. *Random Matrices.* 3 edition, 2004.