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To Referee of Paper 2406.09151v1

SUBJECT: Reply on Report

Higher spin swampland conjecture for massive AdS_3 gravity

Dear Referee,

We thank you for taking the time to read our paper, for the feedback, questions and the much appreciated suggestions.

Please find below answers to the questions in your report as well as the modifications that we plan to implement in the revised version of the paper.

Answer to report and list of modifications

The answers to the three questions are provided in the order they appear in the initial report.

1. **Review on 3D topological massive gravity and the derivation of the mass formula:**

   * **Review of the massive gravity formulation in higher-spin theory**
   In 3D, the pure gravity is known to be topological due to the absence of local degrees of freedom, offering simple settings that allow for tractable studies of gravitational theories, including those coupled to higher spin fields. This pure theory can be extended by incorporating massive degrees of freedom, by deforming the AdS_3 action with a gravitational CS term **(4.3):**

$$\mathcal{S}_1^{\mathrm{GRAV}} = \frac{M_{\mathrm{Pl}}}{2\mu} \int_{\mathcal{M}_{3D}} Tr \left(\Gamma d\Gamma + \frac{2}{3}\Gamma^3 \right) \tag{1}$$

   The modified equations of motion are as follows:

$$G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} = 0 \tag{2}$$

   where the Einstein tensor is given by:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{l_{AdS_3}^2} g_{\mu\nu} \tag{3}$$

   and the Cotton tensor by:

$$C_{\mu\nu} = \frac{1}{2} \varepsilon_\mu^{\alpha\beta} \nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu) \tag{4}$$

   The theory develops a diffeomorphism anomaly given by the difference between the right c_+ and the left c_- central charges as [6, 7, 40, 41]:

$$c_\pm = \frac{3 l_{\mathrm{AdS}_3}}{G_N} \left(1 \pm \frac{1}{\mu l_{\mathrm{AdS}_3}} \right) \tag{5}$$

   The central charges are both positive if and only if:

$$\mu l_{\mathrm{AdS}_3} \geq 1$$

   The case $\mu l_{\mathrm{AdS}_3} = 1$ defines a critical point where gravity is reduced to a chiral theory with only right movers as the central charge of the left sector vanishes $c_- = 0$ and $c_+ = 3 l_{\mathrm{AdS}_3}/G_N$.

   * **The derivation of the mass formula (4.4)**

The mass of the new massive mode of topologically massive spin 2 gravity has been computed in refs [37,38] and is given by:

$$m_{(2)}^2 = \left(\mu + \frac{2}{l_{\mathrm{AdS}_3}}\right)^2 - \frac{1}{l_{\mathrm{AdS}_3}^2} = \frac{1}{l_{\mathrm{AdS}_3}^2}\left(\mu l_{\mathrm{AdS}_3} + 3\right)\left(\mu l_{\mathrm{AdS}_3} + 1\right) \tag{6}$$

It can also be written as:

$$m_{(2)}^2 = \frac{(2-1)}{l_{\mathrm{AdS}_3}^2}\left((2-1)\,\mu l_{\mathrm{AdS}_3} + (2+1)\right)\left(\mu l_{\mathrm{AdS}_3} + 1\right) \tag{7}$$

A possible generalisation of the s=2 formula for higher spin-s field, populating the topologically massive higher spin gravity theories, can be conjectured as:

$$m_{(s)}^2 = \frac{(s-1)}{l_{\mathrm{AdS}_3}^2}\left((s-1)\,\mu l_{\mathrm{AdS}_3} + (s+1)\right)\left(\mu l_{\mathrm{AdS}_3} + 1\right) \tag{8}$$

where for s=2, eq(8) matches eq(7).
We intend to add this complement in section 4, after formula (4.3).

2. **3D gravity + U(1) theory versus 3D higher spin topologically massive gravity:**
Before discussing the differences, we first examine the similarities. For a higher spin gravity theory with $sl(N,\mathbb{R}) \times sl(N,\mathbb{R})$ symmetry, imposing diagonal boundary conditions generates asymptotic symmetries governed by the $u(1)^{(N-1)} \times u(1)^{(N-1)}$ affine algebra [1,2]. The higher spin particles, higher spin versions of the graviton, emerge through composites of the $U(1)$ photons via a twisted Sugawara construction at the boundary. The BTZ black hole solution in this sl(N,$\mathbb{R}$) higher spin gravity theory is therefore analogous to a charged BTZ black hole solution in AdS_3 Einstein gravity with sl(2,$\mathbb{R}$)×sl(2,$\mathbb{R}$) coupled to $U(1)^{(N-2)} \times U(1)^{(N-2)}$ gauge fields. In this case, we introduced massless higher spin degrees of freedom, endowing the BTZ black hole with higher spin charges which correspond to $U(1)$ charges in the diagonal representation.
However, with the inclusion of the gravitational Chern-Simons term, we induce a mass deformation in the theory's geometry. This is evident from the modified equations of motion $G_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0$ having non vanishing Cotton tensor $C_{\mu\nu} \neq 0$ due to the presence of the CS gravitational term. Therefore, unlike the additional gauge charges, the CS gravitational term invokes a mass deformation, yielding massive higher spin degrees of freedom that effect the geometry of the spacetimes and the associated metrics. It becomes necessary to adapt the WGC to these new degrees of freedom that affect the black hole's stability and dynamics.
We will add a paragraph in section 5.2 to highlight this point.

3. **Why have a WGC refinement for higher spin topologically massive gravity?**
   We start by addressing the second part of your question: would the self-interacting particle condensates still form for topologically massive higher spin gravity theories? The condensate forms because of the AdS_3 boundary conditions, which can act as a box that reflects back the emitted particles, enabling them to self-interact in a sub-extremal cloud. Our setting and the physics therein, with the additional mass deformation, is still governed by the choice of the boundary conditions, similar to the standard theory. This suggests that the same arguments for the WGC refinement beyond the mild version are still applicable.
   However, there is another way to prove this need for a refined version for the class of theories we are considering. Under diagonal boundary conditions, one can identify the higher spin symmetry $sl(N, \mathbb{R}) \times sl(N, \mathbb{R})$, with the affine $U(1)^{2(N-1)}$ asymptotically. This shows the presence of multiple U(1) gauge fields at the boundary which further motivates the need for a WGC formulation beyond the mild version involving a single U(1) [1, 2].
   The real question now is which version of the refined WGC should we apply? Usually, in the presence of multiple U(1)s, one can impose the convex hall condition. However, it only requires the emission of a super-extremal vector (a multi-particle state) which would be problematic for the condensate in this case. Our proposed refinement is more natural as it is based on the sl(2,$\mathbb{R}$) representation. Since the emitted super-extremal states with masses $(\text{M}_-^2)_{\Delta,N}$ are closely related to the unitary SL_2 representation $\mathcal{R}_\Delta^-$, the tower of states fulfilling HS Swampland conjecture was then given by the quantum states of $\mathcal{R}_\Delta^-$. We therefore identified the refinement of the WGC as the tower WGC.
   We will expand on this point after equation (4.17).

References

[1] H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo et al., Soft Heisenberg hair on black holes in three dimensions, Phys. Rev. D93 (2016) 101503, arXiv: 1603.04824 [hep-th]

[2] Daniel Grumiller, Alfredo Perez, Stefan Prohazka, David Tempo, Ricardo Troncoso, Higher Spin Black Holes with Soft Hair, JHEP 10 (2016) 119, arXiv:1607.05360 [hep-th].

Best regards,

Rajae Sammani, for the authors