Referee report

We thank the referee for investigating our manuscript and providing useful comments. Below is our response.

1. *The cutting rules derived in this paper relate several correlators with different Bogoliubov coefficients, i.e., different initial conditions. Normally, we think of a theory as fully specified by the Lagrangian and the initial condition. Therefore, correlators with different Bogoliubov coefficients correspond to objects of different theories, rather than different objects in a given theory. This is in contrast to previous cutting rules derived for BD states, where we do have a relation connecting different objects in a given theory. This makes the physical meaning of the non-BD cutting rule a little obscured. I suggest the authors to clarify this point or at least to provide some comments on this issue.*

   Response: We agree with the referee that our cutting rule equations relate some n-point wavefunction coefficient with a particular choice of Bogoliubov coefficients to the wavefunction coefficients where some of the coefficients are interchanged or even complex conjugated. This relates objects in the same theory with different initial conditions. It is indeed a good idea to add a comment to the paper to clarify this point. As to the more philosophical point of "Normally, we think of a theory as fully specified by the Lagrangian and the initial condition" we have perhaps a different point of view. The standard model is still the standard model both at LEP, where we collide electrons, and at LHC, where we collide protons. In other words, our cutting rules are not so different from, say, soft theorems or crossing relations for amplitudes. A soft theorem relates A(hard $\to$ hard+soft) to A(hard $\to$ hard). If I think of all particles ingoing, the first and the second processes have different initial conditions, but most people would say we are constraining the same theory (say QED or GR).

2. *It is often said that the cutting rule is a consequence of bulk unitarity, similar to the optical theorem of flat-space amplitudes. On the other hand, the cutting rules in this work are derived by a direct manipulation of prop-*

agators instead of a unitarity condition (such as the unitarity of the S matrix). Can authors provide more explicit relations between the bulk unitarity and the cutting rule for non-BD theories?

Response: We thank the referee for raising this important issue. Unitarity was used in the step going from the propagator identities, which are valid irrespectively of unitarity (they are just generalizations of $\theta(x) + \theta(-x) = 1$) to the relation among wavefunction coefficients. In this step, we used that the coupling constants are real so they are not affected by the complex conjugation. Notice that we are always working with real fields and therefore, the information of unitarity is encoded in the fact that couplings are assumed to be real. This was also the same place where unitarity was used in arXiv:2103.09832. To clarify this point further, we are eager to add a few comments in the manuscript.

3. *The prescription for removing early time divergences in the paper involves a seemingly time-dependent coupling. Can this time dependence be realized in a more realistic model? Also, does this time-dependent coupling break the scale symmetry of the theory? If not, is there any underlying reason?*

   Response: The time-dependent adiabatic function is just a mathematical procedure to select the correct state at $\tau \to -\infty$. The fictitious time dependence is removed at the end of the calculation where we take $\epsilon \to 0$. The fact that such a time dependence in the Hamiltonian does not affect the scale-invariance of the final results is clear from the proof discussed in Appendix A. In other words, the time dependence of the Hamiltonian is a convenient regulator to specify how one computes an indefinite integral extending to $\tau = -\infty$, where the integrand oscillates. In the appendix, we show that, when we remove the regulator the final result does not depend on what regulator we had used.

We thank the referee again for their useful comments. We hope we have addressed the concerns of the referee and that our manuscript will be seen fit for publication to Scipost.