Reply to Report #2

Quantum rotor in a two-dimensional mesoscopic Bose gas

Michał Suchorowski, Alina Badamshina, Mikhail Lemeshko, Michał Tomza, Artem G. Volosniev

November 26, 2024

> **Referee 2**: This paper is well-organized and clearly written. While some of the results are quite technical, the authors explain them in simple terms. The description of the angulon quasiparticle in real space in terms of the Gross-Pitaevskii equation (GPe) is new and potentially very useful, given that it allows one to capture the physics of non-uniform condensates in the presence of the strong interactions between the boson and the impurity.

We thank the Referee for a positive evaluation of our work. In the reply, we address the comments of the Referee. We hope that the revised manuscript is ready for publication.

> **Referee 2**: The other strength of the manuscript is that the authors consider the system in a harmonic trap so that it can be realized in the experiment, however, this strength also turns out to be its major weakness, because the effects of the trap on the angulon physics are somewhat obscure. In particular, the authors consider the regime where the central density in the trap remains fixed, so for a small number of particles in the system, the potential has to be more confining, and the effects of the trap are important. At the same time, the effects of the trap should be less important as $N \to \infty$. The main analytical result is Eq.(18) and it is derived under the assumption that N is finite, but the trap is turned off. This analytical result agrees well with numerics for a range of $N \gg 1$, but then suddenly breaks down when N becomes larger than some N^*. The fact that such behavior does not seem to follow from the formalism used (from what I understand the result in Eq.(18) should hold for arbitrary $N \gg 1$), makes the whole approach look questionable. Since it is not clear what triggers such a behavior, it is not clear whether the harmonic trap has to do with it.
>
> Just to sum up the above, when the authors talk about different regimes based on the number of particles in the system, it is hard to tell whether this is something inherent to the angulon problem, or some of the regimes are the artifacts of the harmonic confinement.
>
> I think to make the discussion clearer and make the claims of the authors stronger, they should also consider the problem in the absence of harmonic confinement, for example, by putting the system into a box of finite size. Then one can fix the density and study the problem as a function of N and the box size L. If the results in this system agree with the ones in the current manuscript in the regime where $N \gg 1$, then I will accept the validity of the presented results and will be happy to recommend this manuscript to be published in SciPost. Add results/discussion of the angulon physics in the absence of the harmonic confinement and compare with the current results.

Our reply: We thank the Referee for this suggestion. We have performed calculations in a box potential as suggested in the report, see Fig. S1. The plot shows the angular momentum of the bath A as a function of the size of the system in the units of the healing length, ξ. Solid lines show A for a system in a harmonic trap (where R denotes Thomas-Fermi radius), dashed lines show values for the system in a hard wall potential (where R denotes the radius of the potential), and dotted horizontal lines show analytical results for the angulon state. It is clear that while the departure point from the angulon plateau is shifted, the collective excitations in the bath dominate the physics in the limit $N \gg 1$ for both potentials. In fact, the departure from the angulon plateau is more dramatic for a box potential.

Note that in Sec. 3.3, we discuss that the collective excitations of the condensate are expected to have energy lower than the angulon state. Our theoretical formalism (Eq. (18)) does not take these excitations into account, which is the reason why our numerical results agree with theoretical predictions only in the regime where the angulon state is a ground state. Even though collective excitations will appear in any confining potential, the shape of the confinement will affect the exact energy dispersion (and, in effect, the departure point from the angulon plateau).

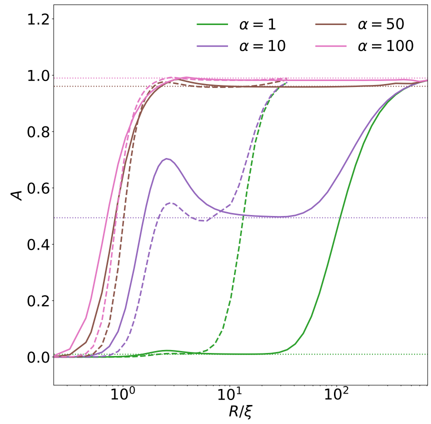

Figure S1: The angular momentum of the bath as a function of the size of the condensate for a harmonic confinement (solid curves) and a box potential (dashed curves).

In the revised version, we added a few sentences to strengthen our discussion. However, we do not add Fig. S1 to the manuscript, as it is already very long in our opinion. Of course, if the Referee believes that this figure must be necessary included, we will reconsider our decision.

Changes to the manuscript:

- We added at the end of paragraph 5 of Sec. 3.3
  Revised manuscript: While energy dispersion of collective excitations depends on the shape of the trapping potential, their emergence is general. Increasing the size of the condensate will lower the energy of excitations for any shape of the confinement. We tested this statement numerically using a box potential (not shown here).
- We change the beginning of the paragraph 2 in section 5.2
  Original submission: Further, we plan to investigate the metastable angulon regime, in particular, the excitations of the Bose gas that define the corresponding ground state of the system.
  Revised manuscript: Further, we plan to investigate the metastable angulon regime, in particular, the excitations of the Bose gas that define the corresponding ground state of the system and how they are affected by the shape of the confining potential.

Referee 2: Finally, the angulon problem is a close cousin of the Bose polaron problem, as was also pointed out by the authors. For the Bose polarons, there is a thermodynamic relation between the number of particles inside the polaronic cloud and its energy, see https://doi.org/10.1103/PhysRevLett.126.123403. Can one expect something similar to hold for angulons?

Our reply: We thank the Referee for this comment, it is well taken. We agree that it is interesting to see if a similar thermodynamic relation is relevant to the angulon problem. Based on our theoretical framework, we established a relation similar to the one in the cited paper, see the new discussion in App. D. However, further work is needed to define the number of particles inside the angulon cloud, which we leave for further studies.

Changes to the manuscript:

- We added relevant paragraphs in Sec. 3.2 and App. D.

Referee 2: Add radial density profiles for weak and strong interactions and comment on how condensate is distorted.

Our reply: Please note that Fig. 10 in App. D shows the densities obtained numerically and analytically in the angulon regime. We believe that these plots give more information than the radial density profiles as the problem is two-dimensional, and angular dependence is crucial to understanding the properties of the system. However, if the Referee believes that such plots would do the paper good, we will include them. Figure S2 show samples of density profiles.

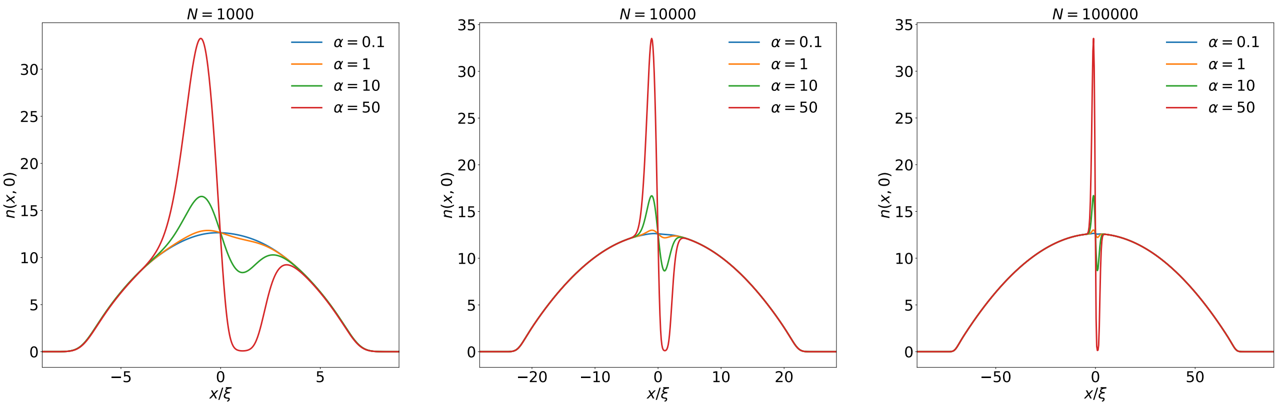

Figure S2: Density of the Bose gas for $y = 0$ for different values of particles and values of the boson-impurity interaction strength.