Reply to Report #1
Quantum rotor in a two-dimensional mesoscopic Bose gas

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Referee 1: I think the journal acceptance criteria are fully met.

We thank the Referee for this encouraging evaluation of our work. Below we address 'Requested changes' of the Referee.

Referee 1: 1. The physical origin of the impurity-boson interaction is unclear. The manuscript suggests the rotor model for the impurity. Are bosons also rotors?

Our reply: We agree that the previous version of the manuscript did not state that bath bosons are point-like particles. Moreover, the motivation behind the choice of the impurity-boson interactions was not included. The revised version contains this information.

Changes to the manuscript:

- We changed the beginning of the first paragraph of section 2.
  Original submission: We study an impurity ('molecule') interacting with N bosons in two spatial dimensions;
  Revised manuscript: We study an impurity ('molecule') interacting with N point-like bosonic particles in two spatial dimensions;

- We changed the last paragraph in section 2.1 and added a relevant citation to *Quantum Theory of Angular Momentum* by Varshalovich D. A. et al.
  Original submission: In general, one can write $W(x,\varphi) = \sum_m R_m(x) e^{im\varphi}$.
  Revised manuscript: In general, one can write the atom-molecule interaction potential in the molecular frame as a Fourier series in terms of angular momentum $W(x,\varphi) = \sum_m R_m(x) e^{im\varphi}$ [15].

Referee 1: 2. On p. 4 we read: "The normalized function Ψ defines the probability of finding a boson at a given position in a molecular frame of reference. It does not depend explicitly on the angle φ_I...", however, coordinates r_i explicitly suggest such dependence. I think this issue should be clarified in the revised manuscript.

Our reply: Following the recommendation of the Referee, we clarified this sentence. In particular, the revised manuscript explains that ψ and function f depend on φ_{I} only implicitly.

Changes to the manuscript:

- We moved the first sentence ("The normalized function ψ defines the probability of finding a boson at a given position in a molecular frame of reference.") to the first paragraph of Sec. 2.2.

- We changed the beginning of the second paragraph of Sec. 2.2.
  Original submission: The normalized function ψ defines the probability of finding a boson at a given position in a molecular frame of reference. It does not explicitly depend on φ_{I}, (...),
  Revised manuscript: The function ψ has only implicit dependence on φ_{I} (via the coordinates $\boldsymbol{r}_i$) (...).

Referee 1: 3. What do the authors mean by the following sentence: Indeed, a strong deformation of the density of the Bose gas requires a number of phonons in momentum space for its description."?

Our reply: We agree that this statement might be unclear. We meant that to describe a strong interaction regime in the momentum space, one would have to include a large number of phonon excitations, which is not trivial.

Changes to the manuscript:

- We changed paragraph 2 of section 4.1.
  Original submission: Indeed, a strong deformation of the density of the Bose gas requires a number of phonons in momentum space for its description.
  Revised manuscript: Indeed, even a basic description of a bound state in momentum space requires a significant modification of the simplest (Fröhlich-like) angulon Hamiltonian [33], complicating the analysis considerably [34].

> **Referee 1**: 4. The breakdown of the angulon behavior at large angular momenta (see Fig. 6) is signaled as a deviation of the rotation energy from the quadratic law. However, at large L the energy $\Delta E_L^{\rm num}$ seems to support a linear dependence. Is it really linear, and is there some physical reasoning for it?

Our reply: This is truly an interesting question. In the current version of the manuscript (see the fourth paragraph of Sec. 4.2) we only speculate that the excitation energy follows $\hbar\omega L$. We motivate it by considering a collective excitation of the non-interacting gas. In the outlook (see Sec. 5.2) we mention that it is our plan to investigate these excitations in more detail. We are working on this question at the moment, and therefore we do not suggest any further modifications to the present manuscript.