We thank the Referee for their positive report and for having raised three important points that helped us to significantly improve our manuscript.

Below please find our response (in black) to all comments, questions, or suggestions raised by the Referee (in blue). We note that in the revised version of our manuscript, we have adopted the latex template for Scipost Physics, as suggested by the journal instructions. This implies that the numbering of the sections changed from Roman to Arabic (i.e., II.C -> 2.3 and so on). In our response, we will consistently use Arabic numbering of sections.

The paper by L. Pisani presents a detailed study of imbalanced Bose-Fermi (BF) mixtures in cold atomic setups. It extends the T-matrix approach, which has been successful for the polaronic theories, to the arbitrary imbalance. The diverse range of observables is discussed in detail.

I have found that the calculations are solid, and the results are very interesting. The analysis of the observables presented is detailed and thoughtful. The paper will impact the field and be noticed by the cold atomic community. I will recommend its publication in the SciPost after the Authors clearly respond to the following points.

We thank the Referee for the above positive comments on our work.

1) The Authors state that "However, since the latter choice gives rise to an improper self-energy contribution, to avoid double counting the -matrix in the normal phase $\Gamma(\mathbf{P},\Omega)$ is used instead, thus yielding. . . ". It is not clear what the improper means here. It looks like the leading contribution (according to the expansion of the Bogoliubov theory) has just been omitted. This approximation needs to be explained in detail.

In the first version of our manuscript, the term "improper self-energy" was used as opposed to "proper self-energy" which, in turn, (see, e.g., A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle System, pag. 105) is a synonym of "irreducible self-energy", i.e., a self-energy insertion that cannot be separated in two pieces by cutting a single particle line. We have verified that the terms "reducible/irreducible" are way more widespread in the literature than "improper/proper".

In the revised version of our manuscript, we thus use only the latter terminology. We thank the Referee for drawing our attention to this point that could have confused readers.

2) The choice of diagrams is motivated by the well-established polaronic regime with the significant BF imbalance, i.e x«1. It seems natural that it would produce reasonable results when x is modestly small. How the choice of diagram is justified at x∼1? For instance, only particle-particle scattering diagrams are taken into account. What about the particle-hole ones? They do not produce bound states but can renormalize chemical potentials, broaden spectral weights, or modify masses. We first would like to emphasize a point that probably was not sufficiently clear in the first version of our manuscript. The recent experiment (Ref. 33), which has confirmed in 3D the predictions obtained by the present choice of diagrams, was not restricted to the case $x \ll 1$ but rather focused mostly on nearly equal densities. We have slightly reformulated the first paragraph of Section 2.2 to make clearer this point.

Independently of these recent experimental results, however, our choice of diagrams is motivated as follows. In the present work, like in previous ones in 3D, we are interested in the competition between boson condensation and the formation of fermionic molecules made of boson-fermion pairs when the attraction between bosons and fermions is varied from weak to strong. Out of all possible Feynman diagrams, we have thus selected a particular class of diagrams (ladder diagrams) that, since the pioneering work by Nozieres and Schmitt-Rink (Ref. 100) on the related problem of the BCS-BEC crossover in two-component Fermi gases, is well known to capture pairing (molecular) correlations in the normal phase (see also Ref. 101 for a review). It is only after the inclusion of this class of diagrams that the superfluid critical temperature recovers the Bose-Einstein condensation temperature in the strong-coupling limit of the BCS-BEC crossover. In addition, for the same contact potential we are considering, ladder diagrams also provide the leading self-energy (established many years ago in Ref. 102) in the weak-coupling limit. They thus provide a sensible scheme to describe the whole BCS-BEC crossover, even in the intermediate coupling region in which fully controlled approximations are not available. The same strategy is then adopted for the present problem, in which we are interested in setting up a theory that is able to describe the progressive formation of paring (molecular) correlations in a Bose-Fermi mixture when the BF interaction is varied from weak to strong. When switching to this problem, the required modification is straightforward in the normal phase: the particle-particle ladder made of the repeated interaction of spin-up and spin-down fermions is replaced by a particle-particle ladder made of the repeated interaction of bosons with (one-component) fermions. When extending the theory to the condensed phase, one has to take into account the possibility that fermions repeatedly interact also with condensed bosons, besides non-condensed bosons. By summing all possible combinations of the repeated scattering of fermions with condensed or non-condensed bosons one obtains the many-body T-matrix in the condensed phase described by the Feynman diagram of Fig. 1. Finally, like in the corresponding problem for the BCS-BEC crossover, when constructing the self-energy of one species, the T-matrix needs to be closed by a propagator of the other species. For the fermionic self-energy in the condensed phase, the bosonic line might be either a condensed line or a non-condensed line. In the first case, however, the many-body T-matrix in the condensed phase $T(\mathbf{P},\Omega)$ needs to be replaced by $\Gamma(\mathbf{P},\Omega)$ in order for the self-energy to be irreducible (and thus avoiding a double-counting of diagrams when inserting the self-energy in the Dyson equation).

We wish, finally, to emphasize that, similarly to the corresponding theoretical approach for the BCS-BEC crossover, our choice of diagrams, besides capturing boson-fermion pairing for strong BF coupling, also reproduces the perturbative results of Ref. 104 in the opposite weak-coupling limit, as explicitly shown in Sec. 4.3. It is thus expected to provide a sensible first approximation to describe the whole evolution from weak to strong coupling. Clearly, we concur with the Referee that the present approximation could be improved in the future with the inclusion of additional diagrams.

We have now added the above comments justifying our choice of diagrams in the revised version of the manuscript. See in particular, the third and fourth paragraphs of Sec. 2.2, the paragraph just above Eq. 7, the paragraph just above Eq. 9, the paragraph just below Eq. 11, the first sentence after Eq. 55, and the last sentence of section 4.3.

3) The described physics has recently been found relevant and fruitful for electron-exciton systems in semiconductor nanostructures. The paper would benefit from discussing whether the obtained

results can be extended to the mentioned solid-state setups.

We now discuss this point in the revised version of our paper. Specifically, in the introduction, we have expanded a paragraph describing these systems that was already present in the first version of the manuscript, while in the conclusions we have added a long paragraph discussing the possible relevance of our investigation to these systems.

For the corresponding changes, see the second paragraph of the introduction, and the paragraph towards the end of the conclusions starting with “It is worth discussing also the relevance of the present investigation...”