

We thank the referee for the comments. Below we address the questions and list the modifications in the revised version of the manuscript.

1. *I think the main result from this paper is the discovery of the asymptotic symmetry for the deformed background (40) and its connections to two-dimensional TTbar deformed CFT. Technically, the authors found an interesting non-local field redefinition to realize the TsT transformation. Then they apply this redefinition to the asymptotic symmetries of the undeformed theory. I would suggest the authors to consider that if the asymptotic symmetry of (40) can be derived directly by imposing proper boundary conditions. I think it should be a very meaningful computation which will reveal the robustness of their previous proposal that deriving the asymptotic symmetry from the worldsheet and also provide a consistent check of the proposed non-local field redefinition.*

Reply: We thank the referee for bringing this up. We have added a new section 5.1 to explain this in detail. Recall that the method of deriving the asymptotic symmetries we proposed in the previous work is to impose boundary conditions on various worldsheet fields and the variation of the equations of motion. In this paper, this method can be directly applied; the boundary conditions are (84) and (85) in the new version. By solving these conditions, we find indeed the symmetries are (88) or equivalently (90). This is consistent with what we obtained using the non-local coordinates.

Revisions: We added subsection 5.1 and some comments below (101).

2. *I wonder if one can consider a flat limit in the present framework.*

Reply: The string background we are studying is asymptotically flat in the string frame, albeit with a linear dilaton. In this sense, the TsT/TTbar correspondence can be regarded as a toy model of flat holography in three dimensions. It will be interesting to try to check if the asymptotic symmetry we found in this paper is related to BMS symmetry. However, it is to be noted that our asymptotic boundary is timelike, which is different from the Bondi gauge. Another subtlety is that our background has a linear dilaton, which means that the metric in Einstein metric is not flat. Due to these reasons, the answer to this question is not immediate, and we would like to leave it for future work.

Revisions: We added some comments above section 5.4 in the revised version.

3. *I wonder if the non-local field redefinition is, in some sense, connected to the cut-off geometry that is dual to TTbar deformation, e.g., 1801.02714.*

Reply: The paper 1801.02714 proposed a holographic duality between the (double trace version of) TTbar deformation and gravity with a finite cutoff, which is also equivalent to mixed boundary condition to AdS₃ in pure Einstein gravity without matter as shown in 1906.11251. On the other hand, the TsT/TTbar correspondence is for the single-trace version of TTbar deformation in which case the bulk geometry is no longer AdS₃. Despite the differences, the stories do bear some similarities. As we briefly commented in footnote 8 on page 23 in the revised manuscript, the non-local coordinates and the vectors in our paper are quite similar to the ones in appendix A of 2212.09768, where a state-of-the-art discussion of asymptotic symmetry for the double trace TTbar holography can be found. We also commented on a subtlety of the choice of the zero mode below equation (68) on page 15. Note that our zero modes are uniquely determined by the compatibility between the symplectic form and the non-local map, see (64),

while the zero modes in 2212.09768 are chosen to make the charges integrable, which are not unique. A more detailed comparison is presented in the following table.

Our conventions	Conventions in 2212.09768
TsT coordinate u	U
nonlocal \hat{u}	u
rescaled \hat{U}	\hat{u}
current j_0	\mathcal{L}
zero mode η_0	$c_{\mathcal{L}}$
$f_{F,\bar{F}}$	f in A.13
$c_{f_{F,\bar{F}}}$	$c_{\mathcal{L}_f}$